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UNCERTAINTY OF COMPLEX SYSTEMS BY MONTE CARLO SIMULATION

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ABSTRACT

Estimating the uncertainty of complex measurement systems by analytical methods is difficult and often the results do not inspire confidence.

Monte Carlo Simulation (MCS) is widely used in business by decision-makers to assess risk. Risk in this context is uncertainty and can equally apply to the uncertainty of a measurement system.

This approach has been used to assess the uncertainty of gas density, calorific value and Wobbe Index and fiscal allocation metering uncertainty. It has also been used to assess the uncertainty of test separators when regarded as multi-phase meters.

The MCS method of estimating uncertainty is fully compatible with the “Guide to the expression of uncertainty in measurement" [1], the uncertainty of a measurement result is usually evaluated using a mathematical model of the measurement and the law of propagation of uncertainty [1] (Para. 3.4.1). The advent of powerful desktop computers enables this technique to be applied to the propagation of uncertainties of most simple and complex measurement systems including many that cannot be found readily by conventional analytical means.

Results of these investigations will be presented to demonstrate the underlying simplicity and reliability of this approach to uncertainty assessment.

The authors consider the universal nature of MCS is well suited to computer implementation with potential to replace conventional methods for determining measurement uncertainty. We hope that the methods presented will be considered in the development of future uncertainty standards.

1 INTRODUCTION

This paper presents Monte Carlo Simulation methods in two worked examples. The first example is a simplified gas export metering system which is compared with conventional uncertainty methods from ISO5168 [2], [3]. Whilst ISO5168 [2] has been withdrawn by ISO because it does not comply with the “Guide to the expression of uncertainty in measurement" [1], it is nevertheless a practical document in the field of flow measurement. ISO5168 [2] has been replaced by a technical report; ISO5168TR: 1998 [3] until a fully compliant document is available. The second worked example is more complex and compares two empirical methods of finding gas density uncertainty discussed in a paper by Jim Watson of NEL [4] with the results obtained using MCS.

The examples used are based on actual projects by the authors, simplified for clarity. One of the authors (Martin Basil) has applied the technique on many client projects mainly concerned with fiscal allocation uncertainty including several meter stream uncertainties. In each case uncertainty was first found by conventional methods and verified using Monte Carlo methods with good agreement in all cases.

The application of the Monte Carlo methods to measurement uncertainty arose from fiscal allocation uncertainty projects in which confidence in the results was not high due to the many assumptions made using conventional analytical uncertainty methods. The availability of a method of assessing measurement uncertainties in relatively complex systems that is:

a) readily understood;
b) easily repeated; and,
c) gives results in agreement with conventional analytical methods,

is the main reason for advancing this method.

2 MONTE CARLO SIMULATION AND ITS USES

How is Monte Carlo simulation used to find the propagation of uncertainties through a measurement system? The measurement system comprises a number of sensors whose outputs are fed into a computation unit that generates the measurement system output (or outputs). We suppose that a definite reference value is applied to the input of each sensor. If we look at the output of one of the sensors at any time, however, it can have one of a range of values that can correspond to the reference value, depending on how the sensor is made, and how well it works.

Using a desktop computer, we can readily generate a realistic distribution of output values for each sensor. We then select at random output values for each sensor and combine them according to the measurement system algorithms. We repeat this as many times as we want. The output of the measurement system will show a range of values for the set of sensor input reference values. We can then analyse that distribution of values using standard statistical techniques to generate the best estimate of the required measurement, and its uncertainty, corresponding to the set of sensor reference input values. We then repeat the process for sets of sensor input values, and generate estimates of uncertainty over the desired operational range of the measurement system.

In a short time (tens of minutes) we can simulate someone carrying out the same process with the real measurement system. However, it is usually quite impractical to carry out this process with a real measurement system.

Monte Carlo Simulation has a history going back to 1873 in which experiments were performed by dropping needles in a haphazard manner onto a board ruled with parallel straight lines to infer the value of π (=3.1416…) from observations of the number of intersections between needle and lines. The first serious application was in 1908 by W. S. Gosset, (who went by the pseudonym of Student), to help him towards the discovery of the correlation coefficient, and in the same year the Student t-distribution. Monte Carlo Simulation methods were developed as a research tool during the Second World War on the atomic bomb to simulate probabilistic problems concerned with random neutron diffusion in fissile material. The theory was developed further from about 1970 to solve a class of problems where there was not sufficient time to find an exhaustive solution as the number of evaluations expanded exponentially.

Today Monte Carlo Simulation is widely used in all areas of activity including business risk analysis, weather forecasting, atomic physics, drilling, manufacturing etc. Metrology is an area in which this technique has not been applied to any great extent and this particularly applies to flow measurement uncertainty. This may be because neither of the two standards for this area, “Guide to the expression of uncertainty in measurement” [1] and IS05168 [2], provides any guidance in the use of simulation methods other than simple numerical methods used to find input/output sensitivity.

Monte Carlo Simulation offers significant advantages over conventional methods when evaluating the uncertainty of measurement systems with the following characteristics:

- Large number inputs and outputs
- Non-linear relationships requiring 2nd order partial derivatives to find sensitivity terms
- Large measurement uncertainties where linear interpolation assumptions fail
- Complex mathematical or empirical input/output relationships
- Input dependencies where the measurement system modifies the action of other inputs
- Untypical input and output distributions that cannot be described in conventional ways

The purpose of this paper is to explain the Monte Carlo Simulation uncertainty method for the benefit of a wider audience. We hope this will also lead to incorporation of these methods in future revisions of the, “Guide to the expression of uncertainty in measurement” [1] and ISO5168 [2], [3] so that a common approach is used to apply Monte Carlo methods to measurement uncertainty.

3 EXAMPLE 1: GAS EXPORT METERING SYSTEM

A simplified gas export metering system is used to illustrate the calculation of uncertainty by MCS. Uncertainty is first calculated by conventional Root Sum Square (RSS) methods and then by MCS. The close agreement demonstrates the fundamental nature of MCS also showing that either method can be used to check the other.

The gas export metering system in Figure 1 comprises two identical streams fitted with orifice plate meters each with a measurement uncertainty of 1% of reading and a flow rate of 10 mmscfd. This example is only for illustration and considers uncertainties as random.

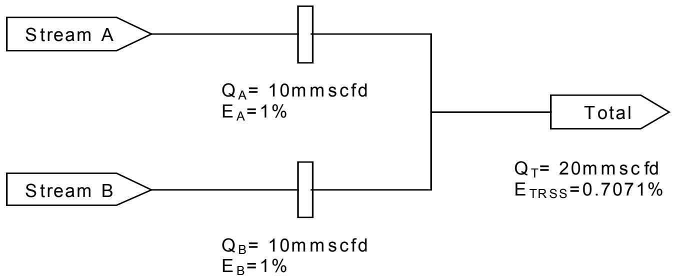

Figure 1 Gas Export Metering System

Uncertainty is found using conventional RSS methods in ISO5168 [2] as follows:

To find uncertainty by MCS the flow rate measurement of each stream is simulated by generating random numbers with a normal distribution that are then applied to a mathematical model of the metering system. The model in this case is simply the sum of the two streams, equation (1). The normal distribution is generated with a mean at the expected measured value of 10 mmscfd and a standard deviation of 0.05 mmscfd is obtained from the measurement uncertainty of 1% of reading with a 95% confidence level.

$$Q_T = Q_A + Q_B \qquad Q_T = 20 \text{ mmscfd} \qquad (1)$$

$$E_{TRSS} = \frac{\sqrt{(Q_A \cdot E_A)^2 + (Q_B \cdot E_B)^2}}{Q_T} \qquad E_{TRSS} = 0.7071\ \% \qquad (2)$$

The simulated stream measurements are applied randomly to the model to give a set of flow rates with a normal distribution. The total flow rate is found from mean of the distribution and the uncertainty with a 95% confidence level is found from twice the standard deviation.

Figure 2 illustrates the simulation of the flow rate measurement of each meter stream input to the model and the resultant distribution. In this case 100,000 trial results were computed in less than 5 minutes. Generally fewer trials, in the region of 5,000 to 20,000, are sufficient for this application however the resulting distribution can appear ragged which may not be acceptable for presentation. The number of trials should be increased in proportion to the number of inputs to ensure most input combinations are properly covered.

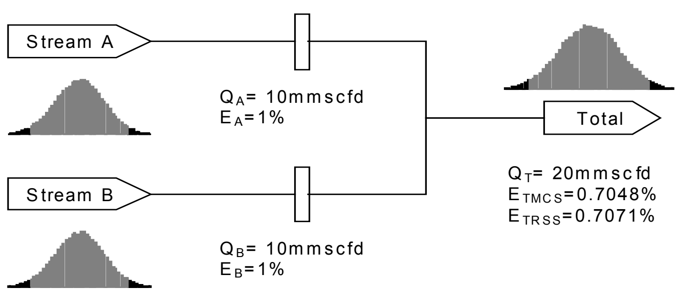

Figure 2 Gas Export Metering System Input Simulation and Model Output Distribution

In this example the MCS uncertainty of 0.7048% of reading is within 0.3% of the uncertainty of 0.7071% of reading found by RSS means.

Figure 2 shows simulated inputs with a normal distribution. Using MCS input distributions may be rectangular, triangular, skewed, discrete values or any other distribution that reflects the nature of the measurement. This can include actual measured data, which is then applied randomly to the model input. Systematic bias errors are particularly difficult to deal with using conventional methods but they can easily be propagated through a model with MCS.

In Figure 3 Stream B flow rate measurement has been given a rectangular distribution with upper and lower limits at 1% of reading. The results again show good agreement between the RSS and MCS uncertainty methods.

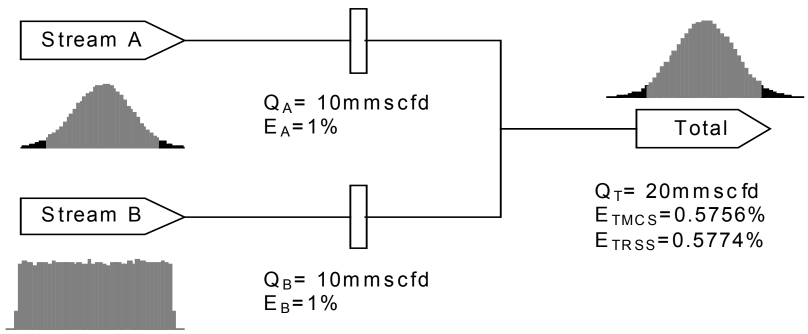

Figure 3 Gas Export Metering System Stream B Rectangular Distribution

To cater for the rectangular distribution the Stream B uncertainty is divided by √3, in accordance with the "Guide to the expression of uncertainty in measurement" [1], before finding the RSS uncertainty.

$$Q_T = Q_A + Q_B \qquad Q_T = 20 \text{ mmscfd} \qquad (3)$$

$$E_{TRSS} = \frac{\sqrt{\left(Q_A \cdot E_A\right)^2 + \left(Q_B \cdot \frac{E_B}{\sqrt{3}}\right)^2}}{Q_T} \qquad E_{TRSS} = 0.5774 \ \% \qquad (4)$$

Results from the examples in Figures 1, 2 and 3 are summarised in Table 1

Stream	Measured Quantity (mmscfd)	Uncertainty (% reading)	A & B Normal Distribution (mmscfd)	A Normal, B Rectangular Distribution (mmscfd)
A	10	1%	9.9895	9.9895
B	10	1%	10.0235	10.0488
Total	20		20.0130	20.0384
MCS Mean			19.9994	20.0002
MCS uncertainty			**0.7043%**	**0.5751%**
Conventional uncertainty			**0.7071%**	**0.5774%**

Table 1 Comparison of Gas Export Uncertainty found by RSS and MCS

4 EXAMPLE 2: AGA8 GAS DENSITY UNCERTAINTY

In this example MCS is used to find gas density uncertainty for two gas mixtures and compare them with the empirical methods discussed in a paper by Jim Watson of NEL [4]. Calorific Value and Wobbe Index uncertainty have also been found by MCS as a further demonstration of this technique.

Gas density can be found from a gas composition, pressure and temperature using the American Gas Association Report No. 8 (AGA8) [5], Detail Characterisation Method. This is a complex calculation based on two equations of state and defined in a FORTRAN computer program for up to 21 gases over a wide pressure and temperature range.

AGA8 has a “Method” uncertainty, tabulated in Table 2, that is dependent on the pressure and temperature of the gas mixture. This uncertainty arises from the experimental data used to develop and verify the standard. These uncertainties apply to the “Normal” range of gas mixtures for all regions and have also been verified for the “Extended” range of mixtures in Region 1.

Region	Uncertainty (% reading)	Temperature Band (ºC)	Pressure Band (Mpa)
1	0.1	-8 to 62	0 to 12
2	0.3	-60 to 120	0 to 17
3	0.5	-130 to 200	0 to 70
4	1.0	-130 to 200	70 to 140

Table 2 AGA8 Gas Compressibility Uncertainty

Using AGA8, the density uncertainty of a natural gas arises from the following sources:

a) pressure measurement uncertainty;

b) temperature measurement uncertainty;

c) gas component uncertainty from:

CO_2, N_2, C1 to C7+	- online gas chromatograph;
H_2S	- analyser;
C8 to C12	- C7+ tail from periodic sample analysis

d) the uncertainty of the AGA8 calculation method.

Conventional analytical uncertainty calls for a calculation of the sensitivity of the gas density to each of the input terms. Sensitivity is found from the partial derivatives of each input with respect to the gas density. In practice this is not possible due to the large number of input terms and the internal complexity of the AGA8 calculation.

Sensitivity may also be found numerically by introducing a perturbation to each input of the AGA8 calculation to find the change in gas density. By combining the input sensitivity with the input uncertainty using RSS, the gas density uncertainty can be obtained, but this approach does not cater for interactions between inputs within the calculation.

Two alternative empirical gas density uncertainty methods that have been suggested in the NEL paper [4] are:

1 General method - each component uncertainty is weighted in proportion to the mole fraction of the component

$$E_{C_i} = E_i \cdot X_i \qquad (5)$$

2 Rich Natural Gas (RNG) - a log rule is used that applies a greater uncertainty to components with a smaller mole fraction.

$$E_{C_i} = 0.1 \cdot \left[1 + \left(\log_{10}(X_i)^2 \right) \right] \qquad (6)$$

The component uncertainties are combined using RSS with the uncertainty of 0.15% due to pressure, 0.1% due to temperature and the AGA8 method uncertainty of 1.0% to find the overall gas density uncertainty:

1 Weighted Component Method

$$E_{\rho gas1} = \sqrt{\sum_i \left(E_{.i} \cdot X_i\right)^2 + E_t^2 + E_P^2 + E_{AGA8}^2} \quad (7)$$

2 RNG Log Rule Method

$$E_{\rho gas2} = \sqrt{\sum_i \left[0.1 \cdot \left[1 + \left(\log_{10}\left(X_i\right)^2\right)\right]\right]^2 + E_t^2 + E_P^2 + E_{AGA8}^2} \quad (8)$$

The gas density is found using MCS by treating the AGA8 calculation as a black box illustrated in Figure 4.

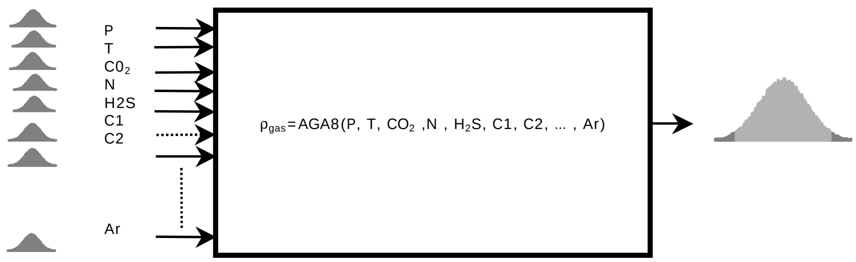

Figure 4 MCS Gas Density

For each input to the black box a range of values is generated with a normal distribution whose mean corresponds to the input value and whose standard deviation corresponds to the uncertainty of the input value at the 95% confidence level. For each trial a value is selected at random for each input and applied to the AGA8 function within the black box and a value of gas density is calculated. Trials are repeated to generate a realistic distribution of gas densities corresponding to the uncertainties of the inputs.

Gas density is found from the mean of the output distribution, which in this case is a normal distribution. The gas density uncertainty at a 95% confidence level is found from standard deviation multiplied by 1.96. This uncertainty due to the input measurement uncertainty is combined with AGA8 method uncertainty using RSS to give the overall gas density uncertainty.

Gas density uncertainty, Calorific Value and Wobbe Index are found for two normalised gas mixtures. Firstly, a natural gas mixture and secondly, the Rich Natural Gas mixture RNG1 in the NEL paper [4]. The empirical methods are used to find the gas density uncertainty and then compared with the uncertainty found by MCS at 100 bar absolute and 45 ºC. Component uncertainty is 0.5% reading for the Weighted and MCS uncertainty method and does not apply to the Log Rule method.

The composition of each gas mixture is shown in Table 3. Gas properties and uncertainty for each gas is summarised in Table 4 for the three uncertainty methods.

Gas (mol%)	N_2	CO_2	C1	C2	C3	nC4	IC4	NC5
Natural Gas	0.92	0.97	93.29	4.29	0.39	0.06	0.03	0.05
RNG 1	5.00	6.00	59.00	10.00	14.00	6.00	0.00	0.00

Table 3 Gas Mixture Composition

Gas	Property	Value	AGA8 Method uncertainty	Uncertainty		
				Weighted	Log Rule	MCS
Natural Gas	Line Density (kg/m^3)	74.603	0.1%	0.52%	0.77%	0.60%
	Standard Density (kg/Sm^3)	0.7291	0.1%			0.45%
	CV 15/15 Real Sup (Mj/Sm^3)	34.67				0.47%
	Wobbe Index (Mj/Sm^3)	50.10				0.26%
RNG 1	Line Density (kg/m^3)	148.967	0.1%	0.39%	0.63%	0.58%
	Standard Density (kg/Sm^3)	1.112	0.1%			0.26%
	CV 15/15 Real Sup (Mj/Sm^3)	49.56				0.30%
	Wobbe Index (Mj/Sm^3)	52.02				0.20%

Table 4 Gas Density Uncertainty Comparison 100% Mixture

The comparison in Table 4 shows agreement for line density uncertainty between the Weighted method and MCS for Natural Gas. Similarly there is good agreement between MCS and the Log Rule method with the RNG for which the Log Rule was designed.

In the case of the two empirical methods the uncertainty due to pressure and temperature had to be estimated by other means whereas this was inherent in the MCS method. The dominant effect of the pressure uncertainty on the overall uncertainty requires particular attention.

The Log Rule method gas component uncertainty is calculated. Agreement between uncertainty results will not be as close for component uncertainties for values differing from the 0.5% used in this example.

In this example the AGA8 method uncertainty is 0.1% and is not a major influence on the overall density uncertainty at line conditions. Where the method uncertainty is higher the influence on the final uncertainty is greater and can be the main source of uncertainty at extreme temperatures and pressures.

In the course of this work it was noticed that when one gas is dominant, such as methane in natural gas, then normalisation of the gas composition leads to:

a) a reduction in the spread of results and consequently the uncertainty;
b) an offset in the final density from the un-normalised result.

The reduction in the spread of results is due to the normalisation process in which all of the components become dependent on each other. Some uncertainties cancel reducing the overall uncertainty. This only becomes apparent when applying MCS and is masked by the simplifications in the other methods discussed, which do not take account of dependencies.

The offset in the final density is due to normalisation when one gas, methane, is dominant. Methane is redistributed over gases with a greater molecular weight without applying a correction for difference in molecular weight. In the following example it is shown that this effect may be significant and requires further investigation into the methods and the need for normalisation.

The initial gas composition of 99.5 mol% is based upon the natural gas described above with a 0.5% mol% reduction in methane. Table 5 on the next page shows the initial gas composition, and the normalised composition with the normalised and un-normalised gas properties and uncertainties.

Input	Temperature deg C	Pressure bar a	Nitrogen mol%	Carbon Dioxide mol%	Methane mol%	Ethane mol%	Propane mol%	n-Butane mol%	i-Butane mol%	n-Pentane mol%	Total Composition mol%	Line Density Kg/m3	Standard Density Kg/Sm3	Calorific Value 15/15 Sup Real Mj/Sm3	Wobbe Index Real(AGA8) Mj/Sm3
Input (Mean)	45.000	100.00000	0.920	0.970	92.790	4.290	0.390	0.060	0.030	0.050	99.500	74.161	0.726	38.458	49.975
Uncertainty (% Input)		0.150%	0.500%	0.500%	0.500%	0.500%	0.500%	0.500%	0.500%	0.500%					
Uncertainty (Units)	0.25000	0.15000	0.00460	0.00485	0.46395	0.02145	0.00195	0.00030	0.00015	0.00025					
Normalised mol%	45.000	100.00000	0.925	0.975	93.256	4.312	0.392	0.060	0.030	0.050	100.000	74.636	0.729	38.652	50.101
											Method Uncertainty % Reading	0.100%	0.100%	0.100%	0.100%
											Monte Carlo Uncertainty %Reading	0.596%	0.441%	0.462%	0.242%
											Monte Carlo inc Method Uncertainty %Reading	**0.604%**	0.452%	0.473%	0.261%
											Normalised Monte Carlo Uncertainty %Reading	0.215%	0.037%	0.020%	0.014%
											Normalised Monte Carlo inc Method Uncertainty %Reading	0.238%	0.107%	0.102%	0.101%
Log Rule Uncertainty %Reading	0.1000%	0.1500%	0.1001%	0.1000%	0.4871%	0.1400%	0.1167%	0.2493%	0.3319%	0.2693%	**Log Rule Uncertainty inc Method Uncertainty %Reading**	**0.760%**			
Weighted Uncertainty %Reading	0.1000%	0.1500%	0.0046%	0.0049%	0.4640%	0.0215%	0.0020%	0.0003%	0.0002%	0.0003%	**Weighted Uncertainty inc Method Uncertainty %Reading**	**0.508%**			

Table 5 Natural Gas Mixture 99.5% Un-normalised and Normalised Results with 0.5% Component Uncertainty

Line density distribution for un-normalised and normalised results in Figure 5 clearly shows a decrease in uncertainty from 0.59% to 0.22%, before applying the AGA8 method uncertainty, due to normalisation. Figure 5 also shows an increase of 0.48 kg/m^3 (0.65%) in line density from 74.16 kg/m^3 to 74.64 kg/m^3, again due to normalisation.

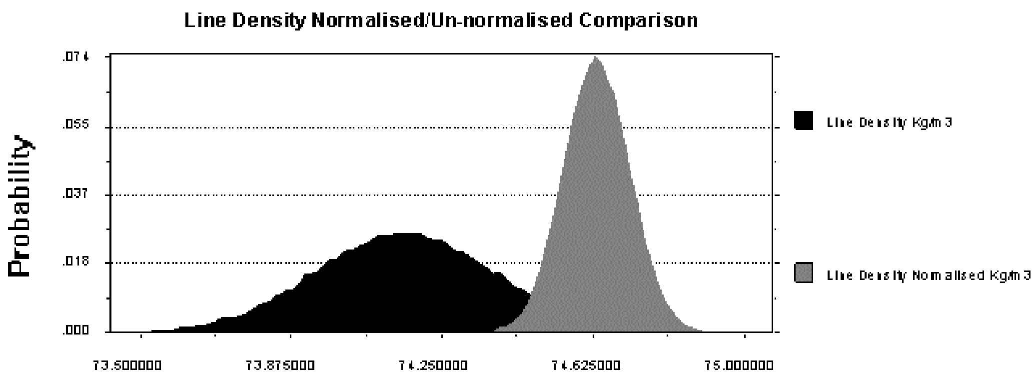

Figure 5 Effect of Normalisation on Line Density

Figure 6 shows the same for standard density with a more pronounced reduction in uncertainty from 0.44% to 0.04% and with an increase in standard density of 0.5%.

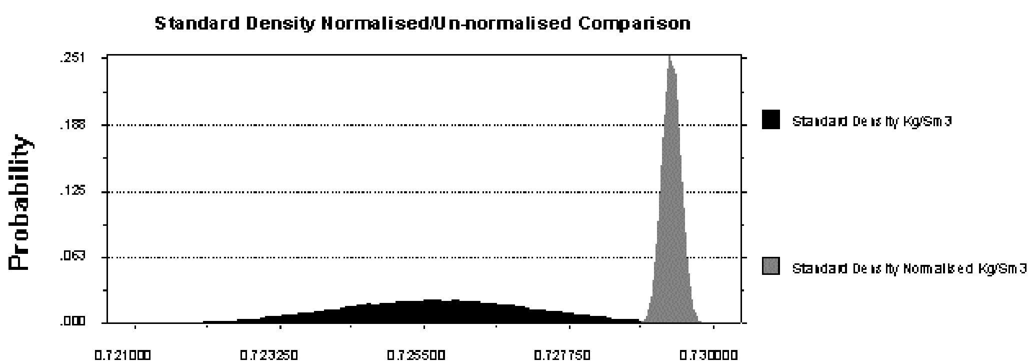

Figure 6 Effect of Normalisation on Standard Density

5 CONCLUSIONS

Monte Carlo Simulation is a suitable method for determining the uncertainty of simple and complex measurement systems. MCS can be readily applied to verify uncertainties found by conventional analytical methods and to serve as a means of detecting errors to ensure confidence in the results.

When estimating the uncertainties of complex measurement systems particular care must be taken in identifying all uncertainty sources and how they propagate through the measurement process. Monte Carlo simulation is a fundamental approach that simulates the true characteristics of a measurement system, and implicitly the propagation of uncertainties through the system.

In the second example MCS allowed the impact of normalisation to be observed highlighting the change in the spread of results and the offset of the mean value. Conventional analytical methods do not provide this degree of insight.

The availability of powerful desktop computers means that engineers can readily apply MCS methods to finding the uncertainty of measurement systems.

6 REFERENCES

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