APPLICATION OF PROBABILISTIC UNCERTAINTY METHODS (MONTE-CARLO SIMULATION) IN FLOW MEASUREMENT UNCERTAINTY ESTIMATION

Martin Basil, **FLOW Ltd**

Christos Papadopoulos, **Cranfield University**

Donald Sutherland, **FLOW Ltd**

Hoi Yeung, **Cranfield University**

INTRODUCTION

Flow is a derived measurement that is invariably obtained from a number of measurements with a functional relationship that must be taken into account when analysing the measurement uncertainty. These relationships are often complex and frequently the results are the input into a larger measurement system.

The conventional uncertainty analysis by the Root Sum Square (RSS) method is often difficult in complex systems and requires approximation at each stage of processing placing serious doubts on the validity of the results. Recent developments in the analysis of uncertainty using Monte Carlo Simulation (MCS) have resolved many of the problems. These include non-symmetric measurement uncertainty distributions, non-linearity within the measurement system, input dependency and systematic bias.

The paper is divided into two parts. The first part of the paper deals with the concept of MCS and its compatibility with the conventional RSS method. The uncertainties of the results of some basic algebraic manipulations, i.e. addition, subtraction, multiplication and division, of two input measured variables which may or may not be correlated are presented to illustrate the how uncertainties are propagated and can be visualised by using MCS technique.

The second part of the paper presents a system for propagating MCS uncertainty developed over the past four years illustrated by some practical applications, covering measurement uncertainty, loss management and allocation uncertainties. Examples from oil, gas and water industries are described. Future development of tools based on MCS will also be outlined.

1 COMPARISON BETWEEN MONTE CARLO AND CONVENTIONAL METHOD

1.1 CONVENTIONAL UNCERTAINTY ESTIMATION

The functional relationship (i.e. measurement model or equation) between the measurand (quantity being measured) Y and the input quantities X_i in a flow measurement process is given by

$$Y = f\left(X_1, X_2, X_3..., X_N\right)$$

The function f includes not only corrections for systematic effects but also accounts for sources of variability, such as those due to different observers, instruments, samples, laboratories and times at which observations are made. Therefore, the general functional relationship expresses not only a physical law but also a measurement process. Some of these variables are under the direct control of the measurement's 'operator', some are under indirect control, some are observed but not controlled and some are not even observed.

The function f is used to calculate the output estimate, y, of the measurand, Y, using the estimates of $x_1, x_2, ..., x_N$ for the values of the N input quantities $X_1, X_2, ..., X_N$:

$$y = f(x_1, x_2, x_3, ...x_N)$$

The methodology of conventional estimation methods can be illustrated using a simple measurement equation with y as a continuous function of x_1 and x_2. y is approximated using a polynomial approximation or a 2nd order Taylor's series expansion about the means:

$$y = f(\bar{x}_1, \bar{x}_2) + \frac{\partial f}{\partial x_1}(x_1 - \bar{x}_1) + \frac{\partial f}{\partial x_2}(x_2 - \bar{x}_2) + W \tag{1}$$

where $\bar{x}_1, \bar{x}_2$ are mean observed values and W is the remainder:

$$W = \frac{1}{2!}\left[\frac{\partial^2 f}{\partial x_1^2}(x_1 - \bar{x}_1)^2 + \frac{\partial^2 f}{\partial x_2^2}(x_2 - \bar{x}_2)^2 + 2\frac{\partial^2 f}{\partial x_1 \partial x_2}(x_1 - \bar{x}_1)(x_2 - \bar{x}_2)\right] \tag{2}$$

As the partial derivatives are computed at the mean values $\bar{x}_1, \bar{x}_2$, they are the same for all $i = 1, ..., N$. All the higher terms are normally neglected with $W = 0$ and, thus, equation (1) becomes

$$y = f(\bar{x}_1, \bar{x}_2) + \frac{\partial f}{\partial x_1}(x_1 - \bar{x}_1) + \frac{\partial f}{\partial x_2}(x_2 - \bar{x}_2) \tag{3}$$

This is acceptable provided that the uncertainties in x_1 and x_2 are small and all values of x_1 and x_2 are close to $\bar{x}_1$ and $\bar{x}_2$ respectively. Additionally, if $f(x_1, x_2)$ is a linear function, then the second order partial derivatives in equation (2) are zero and so the remainder $W = 0$. Both linearity and 'small' uncertainty are prerequisites of conventional method of uncertainty estimation described below.

The standard deviations $\sigma(x_1)$ and $\sigma(x_2)$ are referred to, by the Guide to the Expression of Uncertainty in Measurement (GUM)[1], as the standard uncertainties associated with the input estimates x_1 and x_2. The standard uncertainty in y and can be obtained by Taylor [2]:

$$u(y) = s\,(y) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_i - y)^2} = \sqrt{\left(\frac{\partial f}{\partial x_1}\right)^2 s\,(x_1)^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 s\,(x_2)^2 + 2\frac{\partial f}{\partial x_1}\frac{\partial f}{\partial x_2} s\,(x_1 x_2)} \qquad (4)$$

This equation gives the uncertainty as a standard deviation irrespective of whether or not the measurements of x_1 and x_2 are independent and of the nature of the probability distribution. Equation 3 can be written in terms of the correlation coefficient, $r_{x_1 x_2}$,

$$u(y) = s\,(y) = \sqrt{\left(\frac{\partial f}{\partial x_1}\right)^2 s\,(x_1)^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 s\,(x_2)^2 + 2\frac{\partial f}{\partial x_1}\frac{\partial f}{\partial x_2} r_{X_1 X_2} s\,(x_1) s\,(x_2)} \qquad (5)$$

The partial derivatives are called sensitivity coefficients, which give the effects of each input quantity on the final results (or the sensitivity of the output quantity to each input quantity). The term, expanded uncertainty is used in GUM to express the % confidence interval about the measurement result within which the true value of the measurand is believed to lie and is given by:

$$U(y) = tu(y)$$

where t is the coverage factor on the basis of the confidence required for the interval $y \pm U(y)$.

For a level of confidence of approximately 95% the value of t is 2 for normal distributed measurement. In other words, y is between $y \pm 2s\,(y)$ with 95% confidence.

Coleman and Steele [3], presented a detailed analysis of the subject. The result of a measurement is regarded as only an approximation or estimate of the value of the specific quantity subjected to measurement.

1.2 MONTE CARLO SIMULATION METHOD

With the availability of digital computers, numerical experiments have become an increasingly popular method to analyse physical engineering systems. Simulation is generally defined as the process of replication of the real world based on a set of assumptions and conceived models of reality, Kottegoda and Rosso [4].

Monte Carlo simulation was devised as an experimental probabilistic method to solve difficult deterministic problems since computers can easily simulate a large number of experimental trials that have random outcomes. When applied to uncertainty estimation, random numbers are used to randomly sample parameters' uncertainty space instead of point calculation carried out by conventional methods. Such an analysis is closer with the underlying physics of actual measurement processes that are probabilistic in nature. Nicolis [5], pointed out that in nature the process of measurement, by which the observer communicates with a physical system, is limited by a finite precision. As a result, the 'state' of a system must in reality be understood not as a point in phase space but rather as a small region whose size reflects the finite precision of the measuring apparatus. On the probabilistic view, we look at our system through a 'window' (phase space cell). So the application of Monte Carlo simulation in the uncertainty estimation of different states of a system seems to offer a more realistic approach.

The method can handle both small and large uncertainties in the input quantities. Complex partial differentiations to determine the sensitivity coefficients are not necessary. It also takes care of input covariance or dependencies automatically, Gilks et al[6].

1.3 MONTE CARLO AND CONVENTIONAL METHOD

One of the most important concepts that can be applied to measurement processes is Jensen's (Uncertainty) inequality (see, Kottegoda and Rosso [4]).

$$E(f(X_i)) \neq f(E(X_i)) \qquad (6)$$

It expresses that the end result of a measurement process that combines elementary measurement processes is different from the result of a measurement process of combined elementary measurement process results. In other words, pre-processing of elementary measurement results before we obtain a combined measurement result is not necessarily the same as post-processing the result after the combination of elementary measurements (Papadopoulos [7]). Equality holds when the measurement system is linear.

1.3.1 LINEAR OPERATIONS ($y = x_1 + x_2$; $y = x_1 - x_2$)

Papadopoulos and Yeung [8] reported on the compatibility of Monte Carlo and conventional methods. The effects of correlated and uncorrelated inputs were also examined. It was demonstrated that for linear mathematical operations (i.e. addition and subtraction) the Monte Carlo method give 'identical' results as given in equation (3) and (5). For addition and subtraction, the result of y is a linear function in terms of the inputs x_1 and x_2. Since the first order partial derivatives we all equal to ± 1, the square of which is equal to unity. The second order partial derivatives are both equals to zero, i.e. $\frac{\partial^2 y}{\partial^2 x_1} = \frac{\partial^2 y}{\partial^2 x_x} = 0$. The uncertainty equations are no longer approximations. They are valid for all cases and not limited by conditions such as small uncertainties.

For addition, $y = x_1 + x_2$, both analytical method and Monte Carlo simulation showed that negative correlated inputs reduced the uncertainty of the result. The distribution of the result remains normal. (For the purpose of this paper, the probability distributions of x_i are all normal though Monte Carlo simulation can deal with different distributions). The opposite is true for subtraction, $y = x_1 - x_2$, negative correlated inputs increased the uncertainty of the results.

1.3.2 NONLINEAR OPERATIONS ($y = x_1 x_2$; $y = \frac{x_1}{x_2}$; $y = \frac{x_2}{x_1}$)

1.3.2.1 Multiplication ($y = x_1 x_2$)

By assuming small uncertainties and ignoring the second order effects, the sensitivity coefficients obtained at the mean values $\overline{x_1}$ and $\overline{x_2}$ are:

$$\frac{\partial y}{\partial x_1} = x_2 = \frac{y}{x_1} \text{ and } \frac{\partial y}{\partial x_2} = x_1 = \frac{y}{x_2}.$$

and

$$2s\,(y) = 2\sqrt{\left(x_2 s\,(x_1)\right)^2 + \left(x_1 s\,(x_2)\right)^2 + 2\,r_{x_1 x_2} s\,(x_1) s\,(x_2)}$$

The relative uncertainty of y (for 95.45% confidence) is:

$$\frac{2s\,(y)}{|y|} = 2\sqrt{\left(\frac{s\,(x_1)}{x_1}\right)^2 + \left(\frac{s\,(x_2)}{x_2}\right)^2 + 2\frac{r_{x_1 x_2} s\,(x_1) s\,(x_2)}{x_1 x_2}} \qquad (7)$$

1.3.2.1.1 Uncorrelated inputs

Consider the following simple calculations

$$\overline{x_1} = 6$$

$$\overline{x_2} = 5$$

$$2s\,(x_1) = 0.3$$

$$2s\,(x_2) = 0.1$$

$$2s\,(y) = 2\sqrt{(5 \times 0.15)^2 + (6 \times 0.05)^2} = 1.616$$

$$\frac{2s\,(y)}{|y|} = 2\sqrt{\left(\frac{0.15}{6}\right)^2 + \left(\frac{0.05}{5}\right)^2} = 5.385\%$$

The computed results using Monte Carlo simulation, from one of many single samples of 10000 points are shown in Table 1

. **Table 1 Results of Monte Carlo Simulation** $y = x_1 x_2$ **for** $r_{x_1 x_2} = 0$

	Average	2 x standard deviation	Relative uncertainties (%), (95.45% confidence)
Input			
x_1	6.00000000000000	0.30000000000000	5.00000000000000
x_2	5.00000000000000	0.10000000000000	2.00000000000000
Output			
y	30.00000335838123	1.61483667662676	5.38278831950736

The Monte Carlo output uncertainty average result of 10 samples was, in more than 95% of the cases, within 0.1% of the analytical calculation result. It must be noted that there is a small insignificant bias in the estimated y. The bias corresponds to some residual covariance left due to the single sample used and it is averaged down to zero with a rate $n^{1/2}$ where n is the number of samples. This result showed clearly that Monte Carlo simulation and equations (3) and (5) are fully compatible. (The joint distributions $p(y,x_i)$ of the result is shown in Figure 1 below.

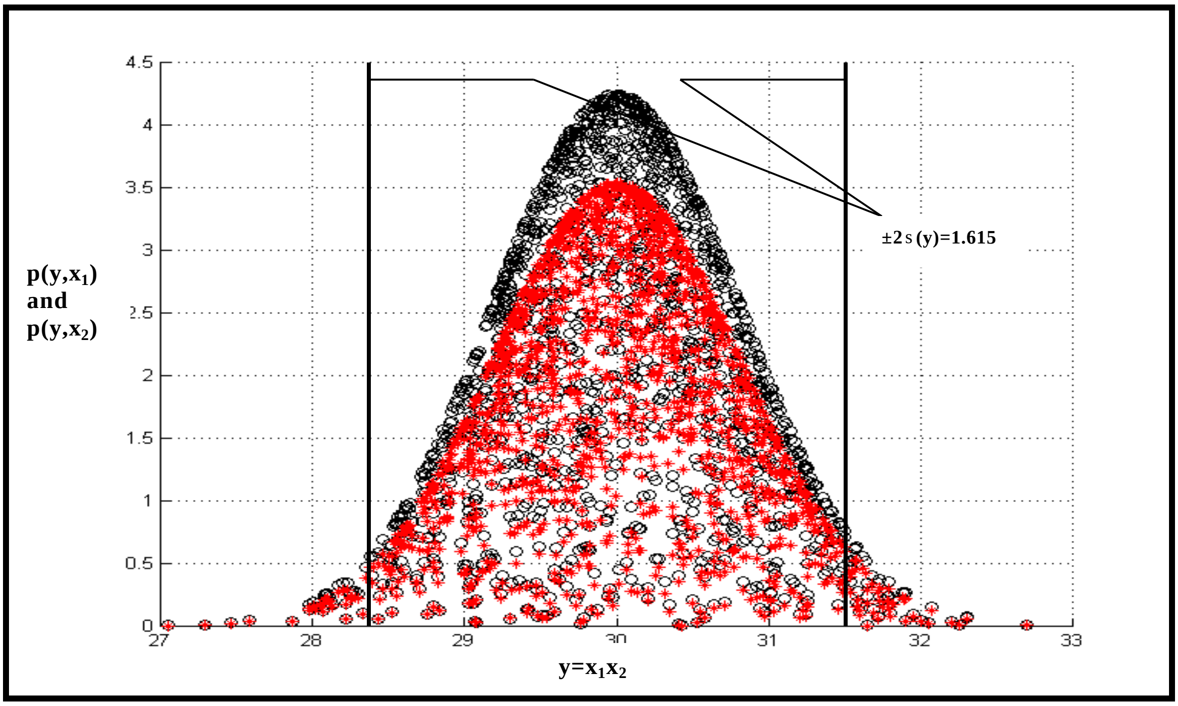


Figure1 Joint distributions $p(y,x_i)$ of $y = x_1 x_2$ **for uncorrelated inputs** $r_{x_1 x_2} = 0$

1.3.2.1.2 Correlated inputs

$\overline{x_1} = 6$

$\overline{x_2} = 5$

$2s\,(x_1) = 0.3$

$2s\,(x_2) = 0.1$

$r_{x_1 x_2} = 1$

$2s\,(y) = 2.1$

$$\frac{2s\,(y)}{|y|} = 2\sqrt{\left(\frac{0.15}{6}\right)^2 + \left(\frac{0.05}{5}\right)^2 + 2\times\frac{1\times 0.15\times 0.05}{6\times 5}} = 7\%$$

Table 2 Result of Monte Carlo simulation $y = x_1 x_2$ for $r_{x_1 x_2} = 1$

	Average	2 x standard deviation	Relative uncertainties %, (95.45% confidence)
Input			
x_1	6.00000000000000	0.30000000000000	5.00000000000000
x_2	5.00000000000000	0.10000000000000	2.00000000000000
Output			
Y	30.007495000000	2.10027879309718	7.00092931032421

Monte Carlo simulation shows that the output $y = 30.0075$ is 0.0075 higher than the product of the average of x_1 and x_2. The difference is not random in this case but represents a systematic 'error'. This is due to the neglect of the residual term W in equation (3). The mean residual term is associated with the inputs covariance (or correlation). The output y can be obtained from equation (1)

$$\overline{y} = \overline{f\,(x_1, x_2)} = \frac{1}{N}\sum_{i=1}^{N}\left[\overline{x_1}\,\overline{x_2} + \frac{\partial f}{\partial x_1}\left(x_{1i} - \overline{x_1}\right) + \frac{\partial f}{\partial x_2}\left(x_{2i} - \overline{x_2}\right) + W\right]$$
$$= \overline{x_1}\,\overline{x_2} + \overline{W} = \overline{x_1}\,\overline{x_2} + r_{x_1 x_2}\, s\,(x_1) s\,(x_2) = 5\times 6 + 1\times 0.05\times 0.15 = 30.0075$$

The contribution of ρ_{x1x2} is clearly displayed in the equation. For positively correlated inputs, there will be a positive contribution. The contribution will be negative for negatively correlated inputs. Perfect positive correlation between the input increased the relative uncertainty of the result from 5.4%, for the uncorrelated case, to 7%. This change is predicted by equation (5) when the correlation coefficient is included. In fact the remainder term W, has no effect on σ.

The standard deviation or uncertainty alone does not reveal that the probability of y is skewed. To illustrate this point, an analysis was carried out with the following parameters:

$$x_1 = 30 \pm 9$$
$$x_2 = 30 \pm 4$$

The large uncertainties were selected in this paper for the purpose of highlighting the effects of correlated inputs on the distribution of y. Figure 2 shows the probabilities p(y,xi) for perfectly positive and negative correlated inputs from the Monte Carlo simulation. It is clear that $r_{x_1x_2} < 0$ results in positive skewness, i.e. long tail on the right. For $r_{x_1x_2} > 0$ the result shows negative skewness, i.e. long tail on the left. Measurement of skewing and flattening (with respect to Gaussian) of the distribution are given by

Coefficient of skewness $g_{1y} = \dfrac{1}{Ns\,(y)^3} \sum_{i=1}^{N} (y_i - \bar{y})^3$

Coefficient of kurtosis $g_{2y} = \dfrac{1}{Ns\,(y)^4} \sum_{i=1}^{N} (y_i - \bar{y})^4$

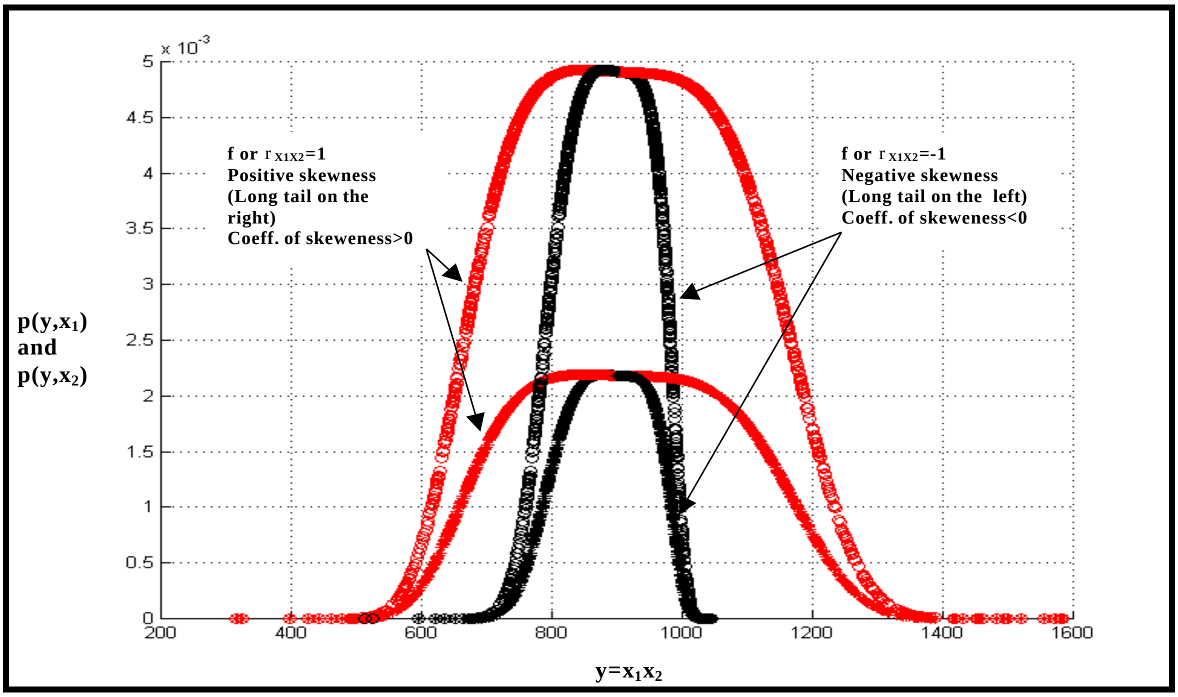


Figure 2 Joint distributions p(y,x$_i$) of $y = x_1 x_2$

for perfect positive correlated input $r_{x_1x_2} = 1$

1.3.2.2 Division ($y=\frac{x_1}{x_2}$ and $y=\frac{x_2}{x_1}$)

For the case of $y=\frac{x_1}{x_2}$, the sensitivity coefficients obtained at the mean values $\overline{x_1}$ and $\overline{x_2}$ are:

$$\frac{\partial y}{\partial x_1}=\frac{1}{x_2} \text{ and } \frac{\partial y}{\partial x_2}=-\frac{x_1}{x_2^2}.$$

The relative uncertainty of y (for 95.45% confidence) is:

$$\frac{2s(y)}{|y|}=2\sqrt{\left(\frac{s(x_1)}{x_1}\right)^2+\left(\frac{s(x_2)}{x_2}\right)^2-2\frac{r_{X_1X_2}s(x_1)s(x_2)}{x_1x_2}} \qquad (8)$$

When compared this with equation (7), it is clear the effect of input correlations for division is opposite to that of multiplication. For positively correlated inputs, the relative uncertainty decreases instead of increases and the opposite is true for negatively correlated inputs.

Simulations have been carried out for $y=x_1/x_2$ and $y=x_2/x_1$ for cases of perfectly positive and negative correlated inputs. The corresponding joint distributions p(y,x_I) are given in Figures 3 and 4. The skewness is much exaggerated. For $y=x_1/x_2$, with the uncertainty of $\frac{Ux_1}{x_1}>\frac{Ux_2}{x_2}$ and x_1 and x_2 positively correlated, the distribution is negatively skewed (with long tail on the left). The opposite is true for $y=x_2/x_1$, the distribution is positively skewed with a long tail on the right.

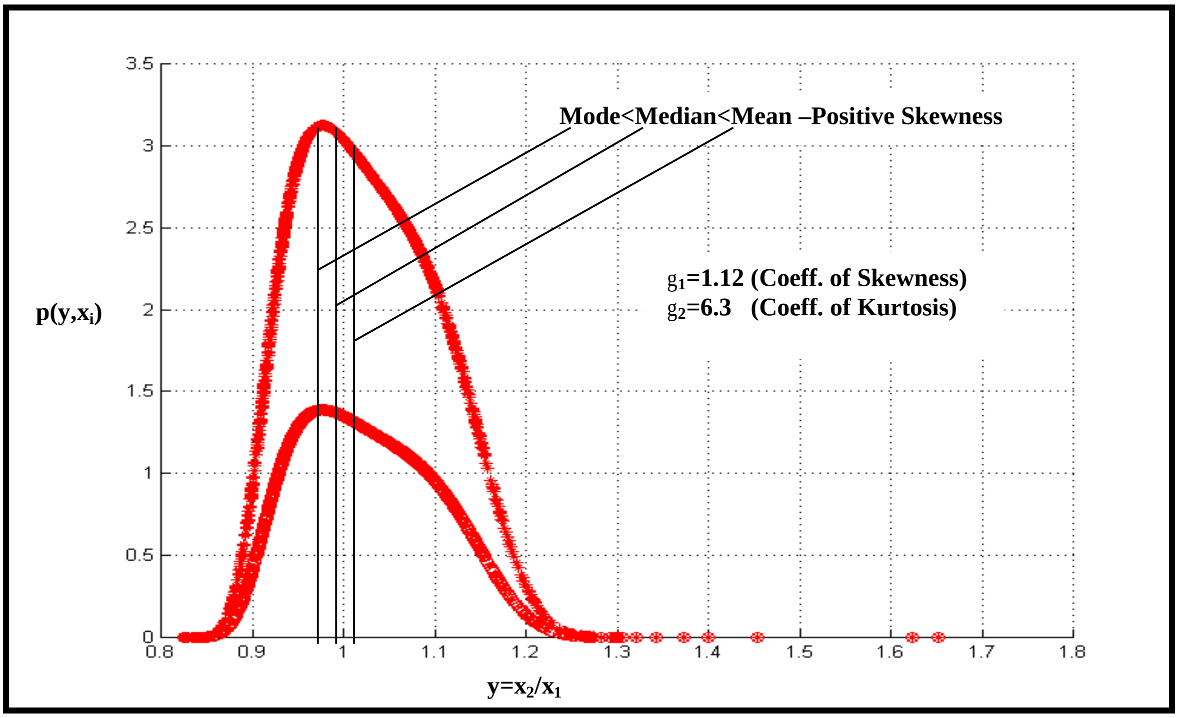


Figure 3 Joint distributions p(y,xi) of $y = x_2/x_1$

for perfect positive correlated input $r_{x_1x_2} = 1$

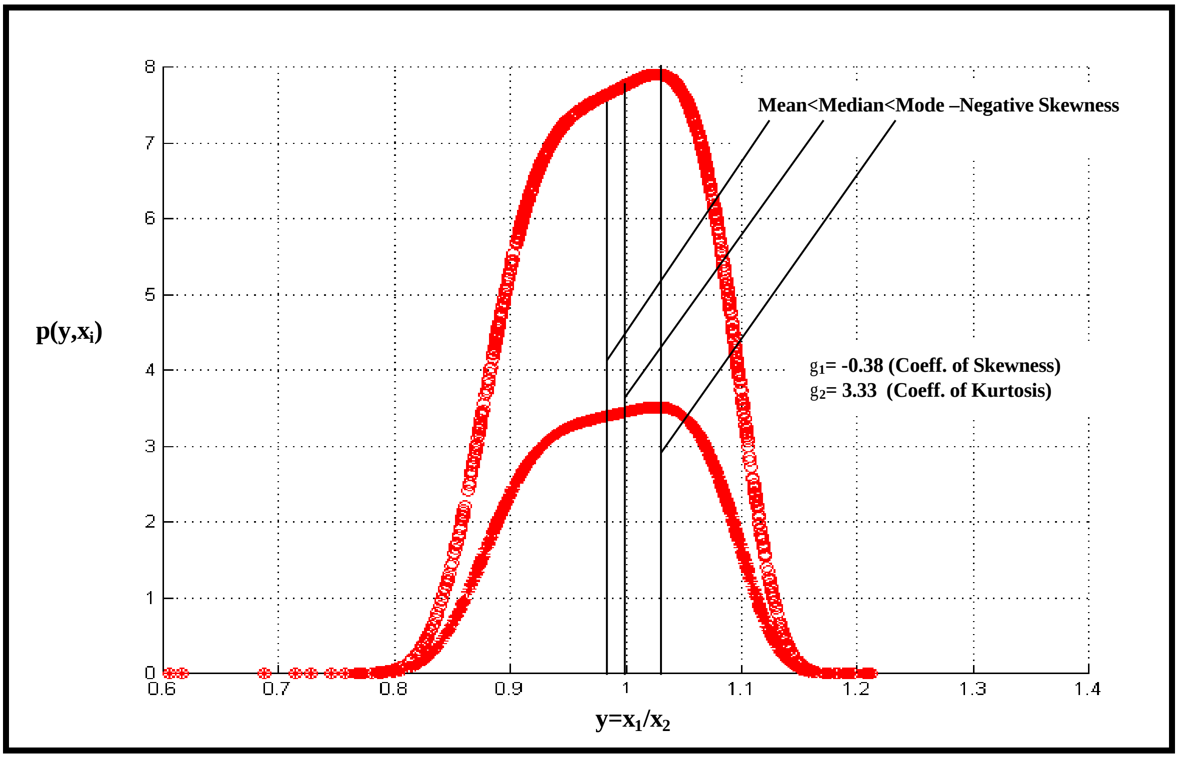


Figure 4 Joint distributions p(y,xI) of $y = x_1/x_2$

for perfect positive correlated inputs $r_{x_1x_2} = 1$

The distributions are skewed and flattened. For $y = x_2/x_1$ with perfect positive correlated input uncertainties, the distribution is positively skewed with a long tail on the right. The coefficient of skewness, $g_{1y} = 1.12 > 0$. The opposite is true for $y = x_1/x_2$, the coefficient of skewness is negative, $g_{1y} = -0.38 > 0$

The examples given above demonstrate Jensen's (Uncertainty) inequality given at the beginning of section 3. In a measurement process during data manipulation, data could become correlated resulting in 'apparent' shift in uncertainty level or values. One such example is the normalisation effects in the analysis of natural gas.

1.4 EFFECTS OF NATURAL GAS NORMALISATION

Normalisation can be seen as an elementary process within the gas energy metering system. Basil and Jamieson [9] reported that when one of the gas constituents is dominant (normally methane), normalisation leads to a reduction is the spread of uncertainty and an offset in the final density calculated from the un-normalised result. These, as it will be shown, are due to the fact that during normalisation process, the 'normalised' methane is no longer independent, but it becomes correlated with the 'mixture'.

To demonstrate the effect of normalisation, an analysis was carried out with gas composition and uncertainties tabulated in Table 3. With the given gas composition, the sum of the concentrations of all the components add to 99.6%. Normalisation to 100% is usually carried out by means of the following equation:

$$X_n = \frac{X_u}{\sum_{i=1}^{k} X_u} 100 \quad (9)$$

where X_u is the un-normalised mole % of substance X;

$\sum_{i=1}^{k} X_u$ is the summation of the un-normalised mole % of all the gas components;

k is the total number of gas components;

X_n is the mole % of substance X after normalisation

This equation is applied for each component separately. The result of Monte Carlo simulation results of normalisation is also tabulated in Table 3.

Table 3 Monte Carlo simulation results for the study of normalisation for ΣX_u=99.6%<100%

Un-normilised Sum of gas components mole % less than 100% - ΣX_u=99.6%, $U\Sigma X_u$=0.095%										
Mol%	**C1**	**C2**	**C3**	**NC4**	**IC4**	**NC5**	**IC5**	**NC6**	**CO2**	**N2**
Unor.	95.5	2.8	0.5	0.05	0.05	0.03	0.02	0.005	0.054	0.6
Norm.	95.875	2.8109	0.502	0.0502	0.0502	0.0301	0.0201	0.00502	0.05421	0.6024
%Unc	**C1**	**C2**	**C3**	**NC4**	**IC4**	**NC5**	**IC5**	**NC6**	**CO2**	**N2**
Unor.	0.1	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Norm.	0.0041	0.096	0.097	0.097	0.097	0.097	0.097	0.097	0.097	0.097

In Figure 5 shows the correlation of the un-normilised mole % of C1 (dominant component) with the summation of the un-normalised mole % of all the gas components (nominator versus denominator of equation (9)). It can be seen that they nearly perfectly positive correlated ($\rho_{C1,\Sigma Xu}$=0.999981).

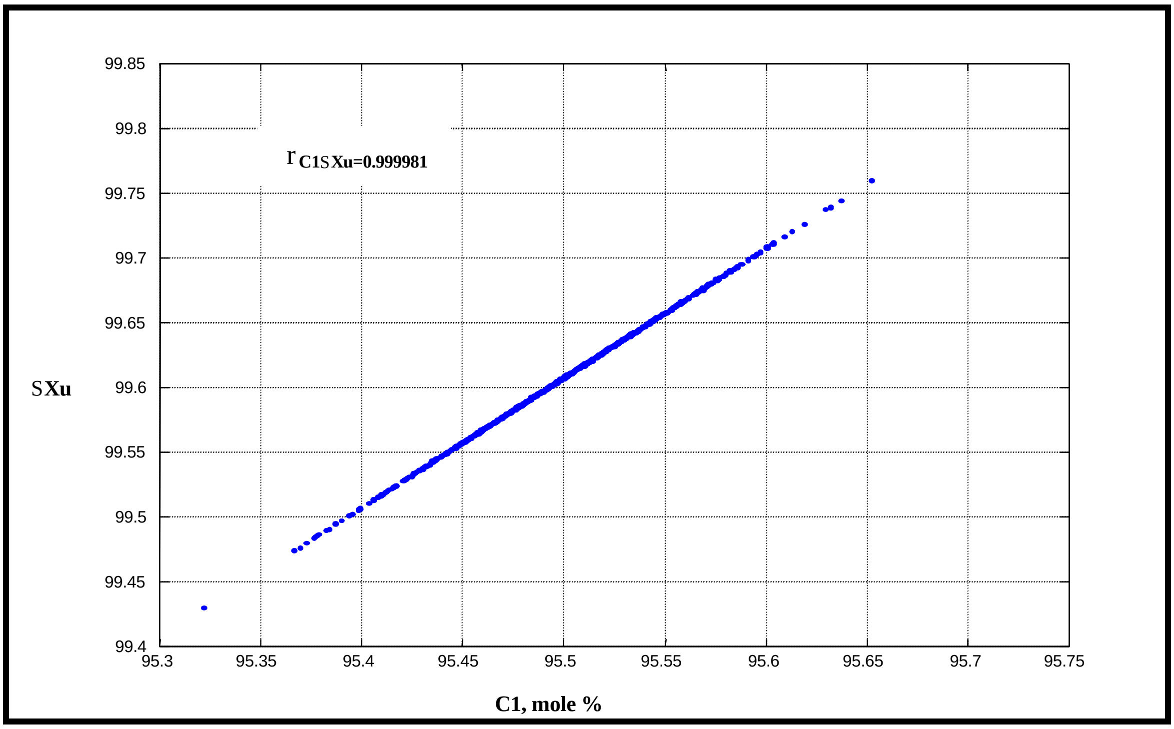


Figure 5 Un-normalised mole % of C1 versus summation of the un-normalised mole % of all the gas components.

The relative uncertainty of the un-normalised C1 obtained, UC1/C1= 0.1/95.5=0.001047 was slightly higher than the respective relative uncertainty of the summation of the un-normalised gas components, $U\Sigma X_u/\Sigma X_u$ =0.095/99.6=0.000954.

It is thus expected that the uncertainty of the normalised mole % of C1 will be much reduced. Assuming perfect correlation, using equation (8), the uncertainty of the normalised C1 is calculated to be 0.0043. This agrees reasonably well with the result of Monte Carlo simulated result of 0.0041.

Normalisation has the opposite effect on the other components. The uncertainty (relative) of all the other components has significantly increased, becoming nearly the same for all of the components and around 0.097%. Again this is as predicted by equation (8).

In terms of the shift of the gas density, that also has been reported by the Basil and Jamieson[9], this is actually due to a respective shift in the calculated mixture molecular weight (molar mass) based on the positively shifted normalised components obtained. Particularly, the last is due to the factor $\frac{100}{\sum_{i=1}^{k} X_u}$, which is multiplied by each of the un-normalised gas components, and is higher than 1 when the summation is less than 100(%), as it was in the above case. Negatively shifted normalised components are obtained (and subsequently a negative shift in calculated molecular weight and in an equation of state based density respectively) when the previous factor is less than 1 (summation more than 100(%)).

1.5 PART 1 CONCLUSIONS

The compatibility of Monte Carlo and conventional methods has been demonstrated. For non-linear functions, errors could be introduced as a consequence of the neglect of the higher order terms. The Monte Carlo method readily takes into account all non-linearity's.

Partial and perfect correlated uncertainties in measurement inputs affect measurement results differently. For linear functions (addition and subtraction), the output is unbiased. Positive correlations increase the uncertainties of the result of addition while negative correlations reduce the uncertainties. The opposite is true for subtraction.

For non-linear functions, the result will have a systematic error when the higher order terms are neglected. Positive correlations increase the uncertainties of the result of multiplication while negative correlations reduce the uncertainties.

The effects of correlated inputs are automatically accounted for by using Monte Carlo simulation. Moreover, the probability distribution of the result can also be visualised.

Monte Carlo simulation is a powerful method and should be more widely used in measurement uncertainty calculations.

2 APPLICATION OF MONTE CARLO SIMULATION UNCERTAINTY METHODS

2.1 APPLICATION OVERVIEW

The first part of this paper demonstrated that the MCS uncertainty method is a reliable alternative means for finding the random uncertainty of a system. It was shown that in addition to finding the random uncertainty, MCS can be used to find bias in uncertainty of systems where the uncertainty of the inputs is large or does not have a normal or equivalent symmetrical distribution. It was also shown that MCS automatically deals with system outputs that have a non-linear functional relationship to the inputs and with systems that have internal dependencies.

This part of the paper presents a system for propagating MCS uncertainty illustrated by some practical applications, covering measurement uncertainty, loss management and allocation uncertainties. Two examples of current project work demonstrate that MCS is a suitable method to find the random uncertainty and systematic bias uncertainty of complex measurement systems. Each example utilises software tools developed by FLOW Ltd over the last four years based on over 50 projects to propagate uncertainty from the measurement devices, through meter stations, to the measurement system outputs. In the first example the allocation uncertainty, and hence financial exposure, is found using MCS for a gas-gathering pipeline in South East Asia. In the second example, MCS is used to find the system balance uncertainty of a water treatment plant to determine the minimum detectable leakage level for loss management.

To further simplify the implementation of probabilistic (MCS) uncertainty models a CAD system is proposed that will enable models to be constructed from a series of calculation blocks that are graphically wired together to propagate uncertainty. The availability of a CAD MCS system will remove the need for specialist skills in the construction of uncertainty models for use in all branches of engineering, science and metrology. The authors believe this will revolutionise the determination of measurement uncertainty in the future.

2.2 COMPLEX MODEL DESIGN

To manage the complexity of large systems, models are designed and constructed as a series of hierarchal components of simple uncertainty models of each measurement device, which in turn are used to construct meter stream, meter station and systems models.

Figure 6 shows a single gas orifice plate measurement stream illustrating all stages of calculation including configuration, initialisation and simulation. This particular element, which is described as a “Generic Uncertainty Model” (GUM), may be configured with switches for most fiscal gas orifice measurement installations, as a standalone model to find the measurement uncertainty of a single stream and as a plug-in component for larger systems.

Initial values are calculated for all measurements that have an uncertainty dependency such as the differential pressure transducer. The uncertainty at flowing conditions is found based on the transducer characteristics, calibration methods and calculation methods using traditional RSS or MCS methods.

The initial values and the uncertainty of each measurement are used to randomly generate trial values with a normal distribution for each simulation input, which are then applied to the flow calculation based on the selected configuration. In this case, configuration concerns the calculation methods used to determine standard density, line density and calorific value. For example, line density may be found with a gas composition from a gas chromatograph with AGA8 or a gas densitometer or using an RD analyser (Relative Density) with PZT correction to line conditions.

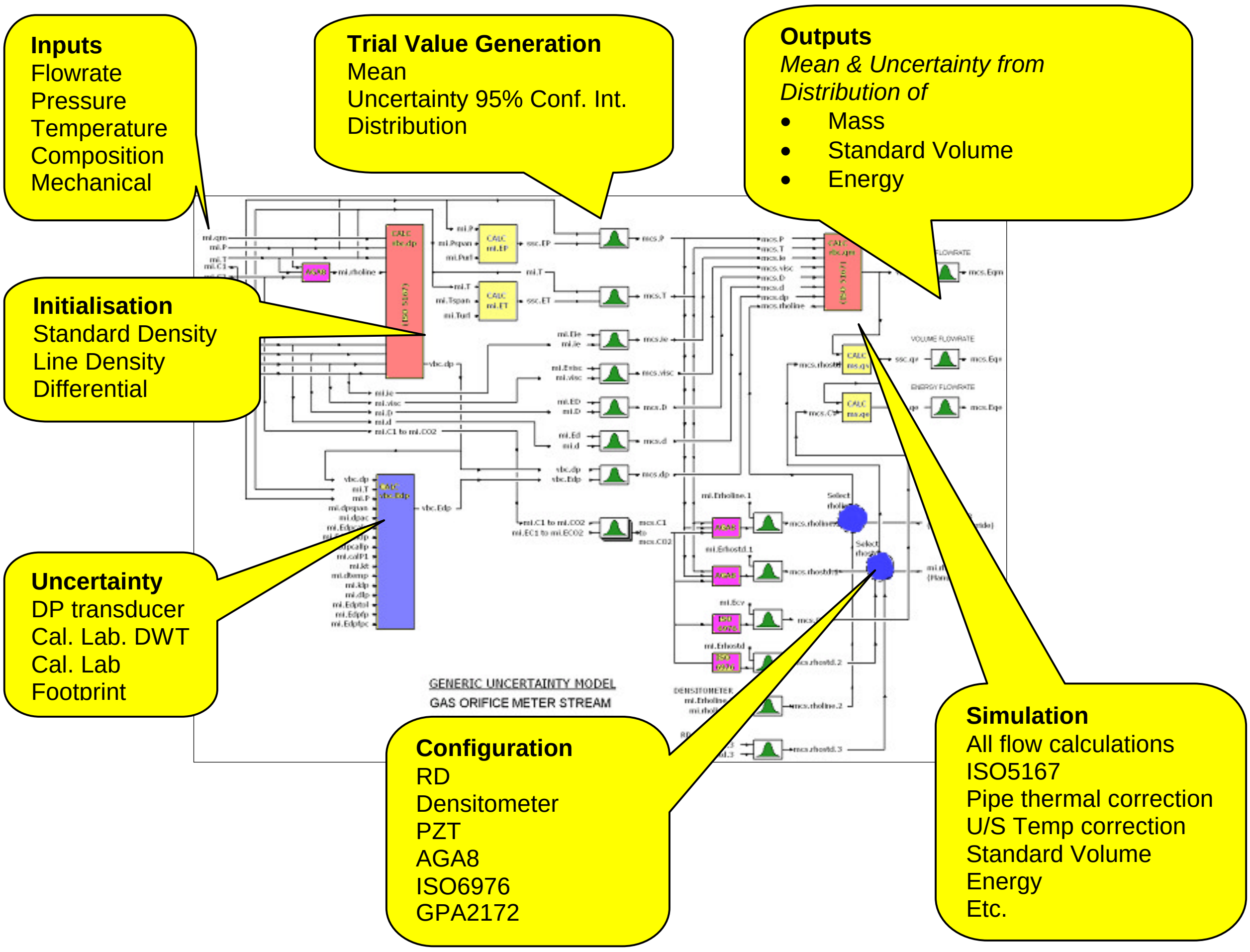


Figure 6 Fiscal Gas Orifice Plate Meter Generic Uncertainty Model (GUM)

Figure 7 shows the GUM in use as a plug-in component, as described by Coughlan, Basil and Cox [10], for a multi-stream installation to find meter station uncertainty, which is in turn plugged in to the pipeline system to find the production allocation uncertainty for each pipeline entrant.

This approach deals with the complexity of the system by assembling small manageable elements starting with the measurement devices. At the lower levels, it is possible to use generic models of measurement devices and standard measurement configurations. This can be extended to higher levels where standard calculation methods are used, for example proportional mass component allocation. The challenge is to produce configurable standard models with sufficient flexibility for most accepted configurations

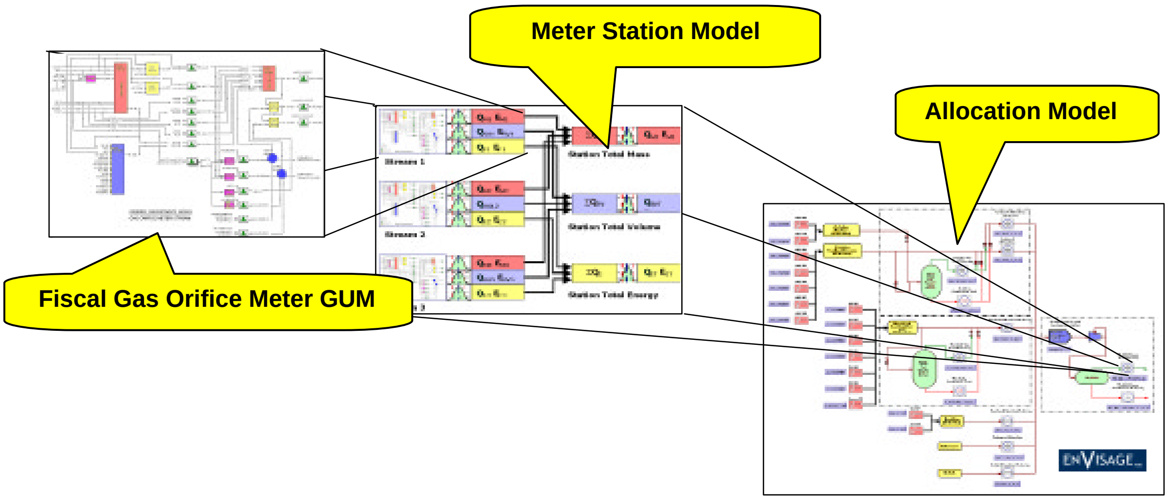


Figure 7 System Uncertainty Model

The difficulty with the hierarchal approach illustrated in Figure 6 and 7 is that even though the model is constructed from separate parts many of which are reusable the model remains a single entity. If any single input to the system changes the entire simulation must be rerun and if the model is constructed using a desktop application such as Excel it quickly becomes unwieldy restricting the maximum size of the model.

2.3 MCS SOFTWARE MODELLING TOOLS

A barrier to the widespread adoption of “Probabilistic Uncertainty Methods” (Monte Carlo Simulation Uncertainty) has been the availability of suitable software to construct models of complex measurement systems. Commercially available MCS software such as @RISK and Crystal Ball are geared towards forecasting risk utilising techniques based on standard deviation limits rather than 95% or 99% confidence intervals. These and other products all require the model to exist as a single entity within a desktop application, usually Microsoft Excel. This approach does not allow for connections between separate models limiting the maximum model size and the entire model simulation must be executed each time any input to the measurement system changes.

Sutherland and Basil [11], showed how by building models up as a series of asynchronous elements very large models can be constructed that may be scaled to any level of complexity. During simulation, each element runs on one or more processors to complete the simulation in the minimum time to enable real time simulation. The database also enables operational scenarios to be saved or retrieved and the construction of time based uncertainty models. With time based modelling, the uncertainty of a measurement system with widely varying inputs and uncertainty can be found over a regular or irregular time interval.

Figure 8 illustrates the use of a database’s with each model to loosely couple the separate model entities. Each database holds the model trial values from the simulation results and the trial value statistics including mean and uncertainty. If a traditional RSS uncertainty model is used in place of an MCS uncertainty model only the database statistics are required. Links to each subsequent model level may use the statistics to generate trial values for input to next stage or the trials values may be resampled using a uniform random number generator.

If the next stage of processing requires more trial values than have been generated at the previous level the database trial values are sampled repeatedly (over-sampling). Under-sampling is also possible however it is not recommended except where the previous level has sufficient additional trials to ensure that the distribution is representative for the reduced number of trials. If all models have the same number of trial values the values may be resampled sequentially because the trial values are randomly distributed in each database.

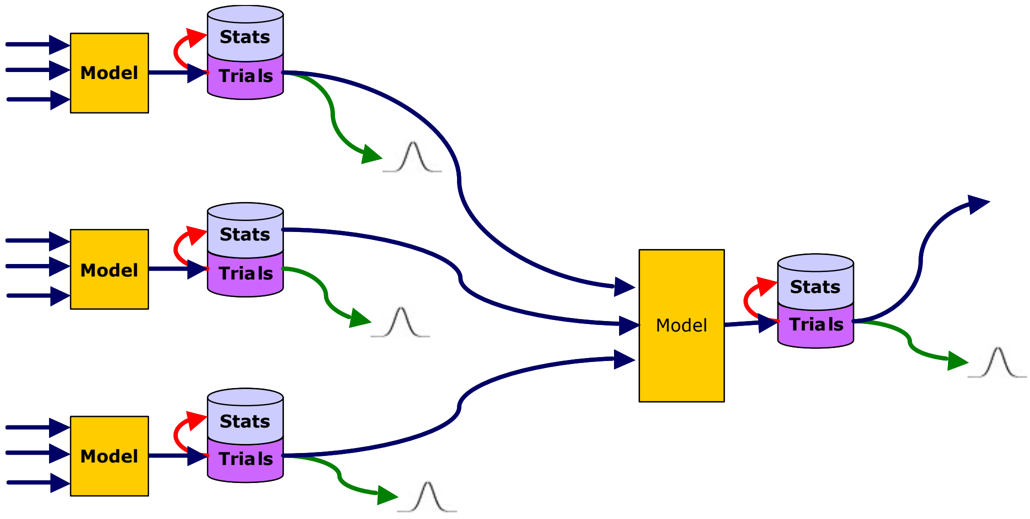


Figure 8 Asynchronous Database Linking of Uncertainty Models

Figure 9 shows the use of scenarios for each model to investigate operational cases such as Minimum, Normal and Maximum for each meter station and then at a subsequent level select combinations of for input to the next model.

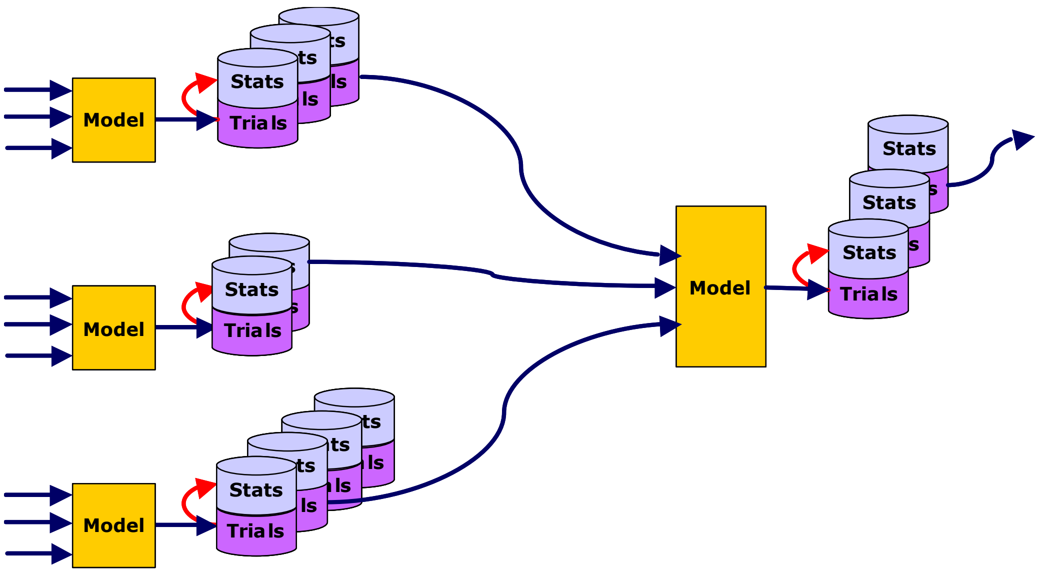
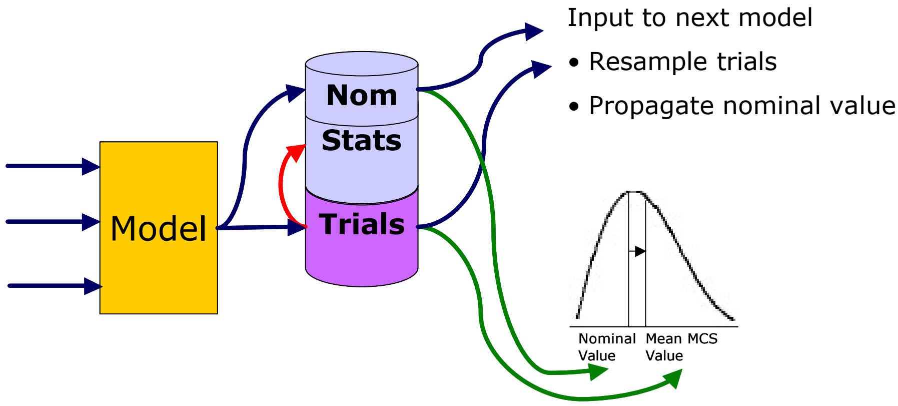


Figure 9 Scenarios for Operational Cases and Time Based Uncertainty Modelling

With time based systems the database is synchronised at each level to create a scenario for each time interval. If the number of trials is the same for all models the final result database values may be summed to integrate the distribution over any time interval. This is useful where there is large variation in the measured values leading to a variation in measurement uncertainty over a relatively short period of time. By integrating the trial values the overall weighted value of the measured variable is found over the chosen time interval.

The trial values and the statistics in the database provide information about the model output at each level. This information includes the distribution of the trial values, the distribution mean, standard deviation, kurtosis, skewness and the calculated model nominal value based on the model input nominal value. In an allocation system this could be the reported allocation quantity so that the model may be used to find final values in addition to the final value uncertainty. Figure 10 illustrates how the calculated nominal value may also be used with the mean of the distribution to find systematic bias at any level in the system.



Bias = Nominal Value – Mean MCS Value

Figure 10 Propagation of Bias

2.4 APPLICATION EXAMPLES

2.4.1 GAS ALLOCATION SYSTEM

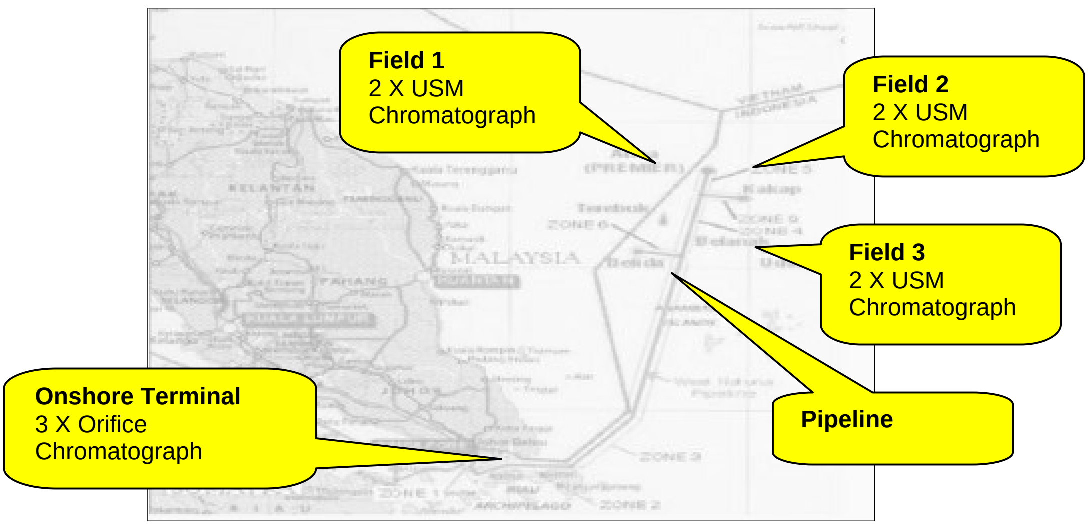


Figure 11 Gas Gathering Pipeline System

An allocation and uncertainty model was supplied to model the gas gathering pipeline system illustrated in Figure 11. Three fields feed the pipeline through identical measurement systems comprising two multi-path ultrasonic meter streams (duty and standby) with an online gas chromatograph. Gas at the receiving terminal is measured with a three-stream orifice system (two duty and one standby) with an online gas chromatograph. The measurement system calculates the real dry gross heating value in accordance with GPA2172 and GPA 2145 to find the energy flow at each pipeline entry point and the onshore terminal receiving point. The energy flow at the receiving terminal is then simply allocated in proportion to the energy flow at each pipeline entry point for accounting purposes. The standard volume for each measurement station is also found and allocated proportionally for reporting purposes.

The four measurement systems are constructed as separate models each with a database to store trial values from each simulation run. These are connected via the databases to an allocation model that performs the following calculations for each field.

Energy allocation:

$$E_{FieldAllocation} = \frac{E_{FieldMeasured} \times E_{Ter\min alMeasured}}{\sum_{Field} E_{FieldMeasured}}$$

Volume allocation:

$$V_{FieldAllocation} = \frac{V_{FieldMeasured} \times V_{Ter\min alMeasured}}{\sum_{Field} V_{FieldMeasured}}$$

Supervisory software schedules the simulation of each measurement model followed by the allocation model to find the energy and volume allocation for each field.

The model was supplied for first gas in January 2001 to allocate daily gas energy production and provide a quality indicator for each field based on current operating conditions. This model will shortly be modified to automatically retrieve data at 15-minute intervals from a Fisher Rosemount Delta V system linked to PI (Process Information) database with OPC (OLE for PC) client software. The allocation and uncertainty are then calculated and the results handed off to the PI database with OPC server software. Other systems then extract the allocation and uncertainty information from the PI database for reporting and accounting.

2.4.2 WATER TREATMENT PLANT SYSTEM BALANCE

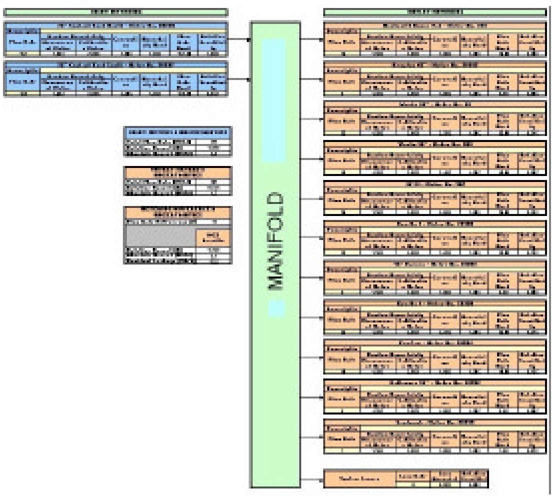


Figure 12 Water Treatment Plant System Balance

A system balance model was developed to find the system balance uncertainty due to the measurement uncertainty of the inlet and outlet meters to a water treatment plant. The system balance uncertainty limits provide the minimum detectable leakage with a 95% confidence interval. In other words if the difference between the inlet and outlet flow rates is less than these limits it is not possible to determine whether the difference is due to leakage or measurement uncertainty. However if the difference is greater than the system balance uncertainty then the amount that is greater than the limits is due to leakage or an equipment failure.

This approach can be extended to include leakage prediction with an uncertainty to compensate for expected leakages. By comparing the expected loss uncertainty with the measured difference short-term changes in the measured rates can be rapidly identified. The expected loss uncertainty may be used to develop norms for pipelines based on the recent system balance history.

The model comprises two inlet meters and eleven outlet meters. The uncertainty for each meter is found from a ten-point look up table with linear interpolation. All the meter uncertainties are applied to the system balance calculation using MCS to find the system balance and uncertainty.

Water usage varies significantly during the day and the meter uncertainty increases significantly at low flow rates therefore the uncertainty and system balance is required for the end of each day, month and annually. By integrating the system balance uncertainty trial values for the required period the overall uncertainty can be found for the period. Trial values are integrated by summing the values for each database entry sequentially and adjusting the summed result for the time period to produce. The uncertainty is found from the summed distribution.

2.5 PROPOSED MCS CAD SYSTEM

The current MCS software tools are at the third stage in a planned development resulting in a graphical CAD system for engineers, scientist and metrologists to develop their own uncertainty models. The current MCS software simplifies and standardises in-house development of uncertainty models but requires the involvement of engineers and programmers with a thorough understanding of uncertainty. The proposed CAD system will remove the need for the direct involvement of programmers and reduce the level of understanding of uncertainty to that of the end users discipline.

The objective of the CAD uncertainty system is to introduce MCS to a much wider audience and simplify the application of uncertainty in all areas of measurement.

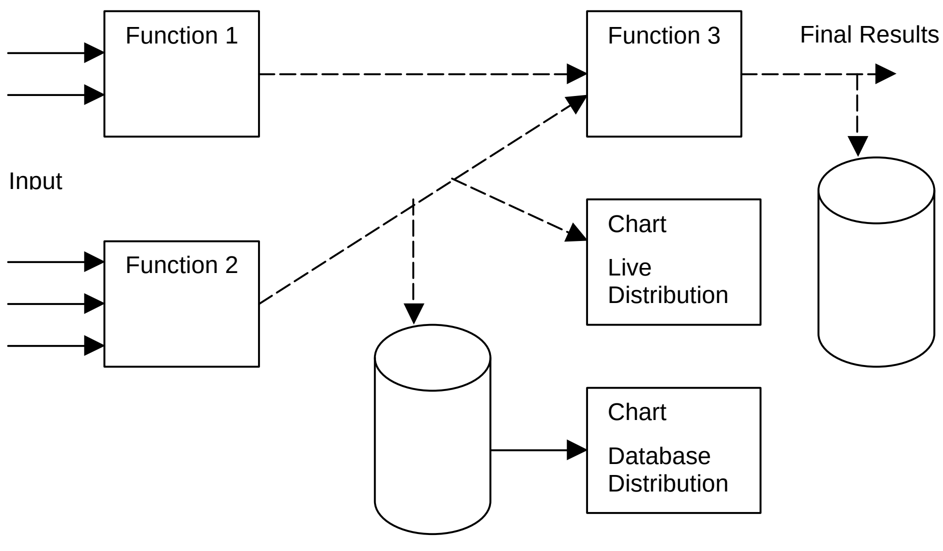


Figure 13 illustrates a possible CAD MCS implementation.

2.6 PART 2 CONCLUSIONS

The practical application of MCS software tools to propagate uncertainty through complex measurement systems has been successfully demonstrated on a number of projects. These include systems with a variety of scenarios and time based systems.

Decoupling models using databases allows scaling up to any level of complexity and enables the use of parallel computers to speed up the processing of results for real time applications.

Generic Uncertainty Models (GUM) mimic the behaviour of measurement devices, transducers and measurement systems to simplify the construction of complex uncertainty models.

Propagating the nominal calculated value with trial values enables the measurement uncertainty model to become the measurement system, delivering both the final values and uncertainty.

By integrating (summating) trial values over a time interval the uncertainty can be found for any period without modification to the transient uncertainty model.

The database statistical data provides a snapshot of all the intermediate and final values and uncertainty results of the measurement system.

The availability of CAD MCS software tools will remove the need for specialist skills in the construction of uncertainty models in all branches of engineering, science and metrology.

The authors believe that MCS will revolutionise the determination of measurement uncertainty.

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