SOLV™

Flow Measurement Uncertainty Assessment

by

Martin Basil, SOLV Limited

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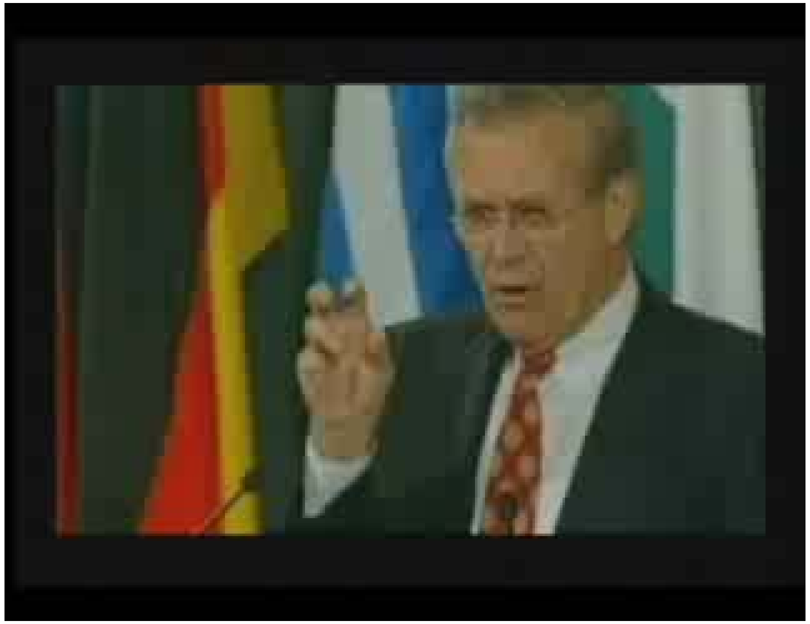
Overview

- Uncertainty
  - definition, standards
  - confidence interval, distribution
  - non-linearity and bias
- Combination of Uncertainties
  - "How long is a piece of string"
  - Quadrature by partial derivative and perturbation
  - (MCS) Monte Carlo Simulation
  - Pipeline Allocation example
- Applications
  - Complex fluid property methods
  - Propagation of uncertainty
- Uncertainty; a philosophical point of view

Known’s & Unknown's



1. There are known known's; there are things we know, that we know;
2. There are known unknowns; that is to say, that there are things we now know, we don’t know;
3. But there are also unknown unknowns; there are things we do not know, we don't know.

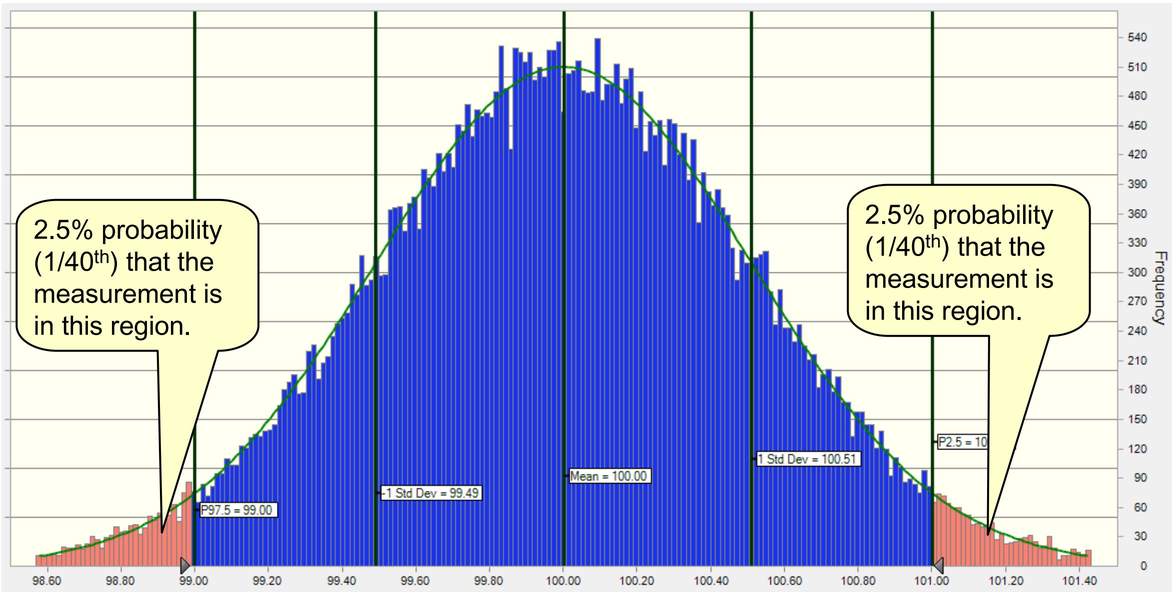
Video: Former US Defense Secretary, Donald Rumsfeld presenting his uncertainty philosophy at a White House press briefing

Standards

- ISO Guide 98: 1995 *"Guide to the expression of uncertainty in measurement"* ; known as the GUM, the over-arching uncertainty standard adopted by ANSI, BSI, OIML and others to which all ISO and OIML uncertainty standards must comply.
- ISO5168: 2005, *"Measurement of fluid flow – Procedures for the evaluation of uncertainties"*; specific to flow measurement, the latest update conforms to the GUM.
- API MPMS Chpt. 13.1: 1985 *"Statistical Aspects of Measuring and Sampling – Statistical Concepts and Procedures in Measurement"*, currently under review.
- ISO Guide 98/DSuppl 1.2, *"Propagations of distributions using a Monte Carlo method"*, supplement to the GUM covering the use of MCS (Monte Carlo Simulation) for uncertainty analysis.

Definition

- Uncertainty is defined as the interval within which 95% of the values are expected fall for a given measurement.
- Normal (Gaussian) distribution shown has a mean of 100 with an uncertainty of ±1.00% OMV (Of Measured Value) with a 95% CI (Confidence Interval).
- 95% CI found from twice the Standard Uncertainty which is the standard deviation of the measurement samples for a Normal distribution.
- Type A Uncertainty   Found by statistical sampling and analysis
- Type B Uncertainty   Found by other means with an assumed probability distribution



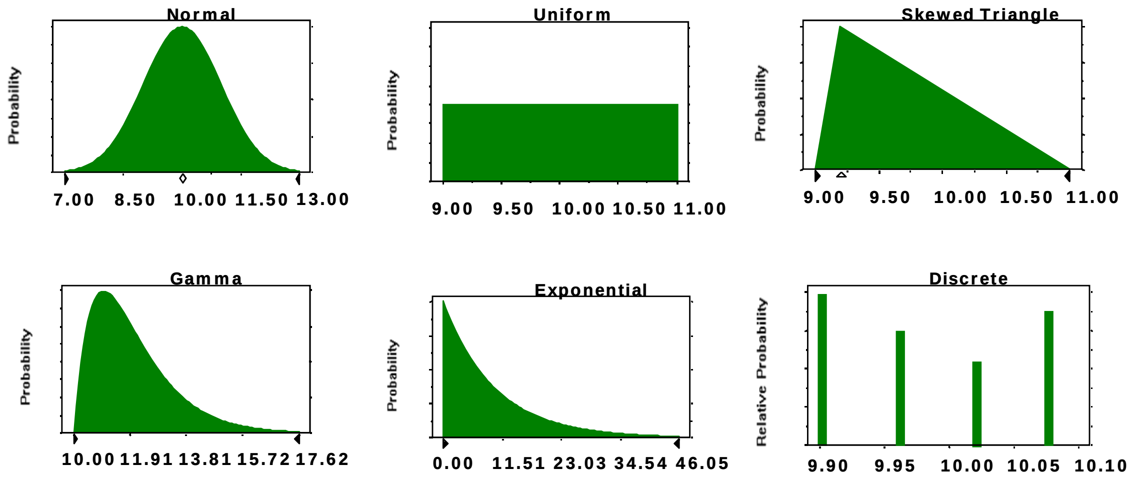
Confidence Interval

- If CI's such as 90% or 99% are used the CI must be stated with along with the uncertainty.
- If the CI is not stated it is taken to be 95%.
- Uncertainty can be adjusted to any CI by multiplying the Standard Uncertainty (standard deviation) by the CF (Coverage Factor):

CI	CF
68%	x1.00
90%	x1.65
95%	x1.96
99%	x2.58

- These factors only apply to a large number of random samples. With a population of 5 samples the CF for a 95% CI increases to 2.57.

Distributions



- Most measurements have a Normal distribution.
- A thermometer scale has discrete steps so the actual temperature falls between two points with an equal probability (uniform distribution).
- A Uniform distribution has a CF of $\sqrt{3}$ (1.72) at a 95% CI.

Central Limit Theorem

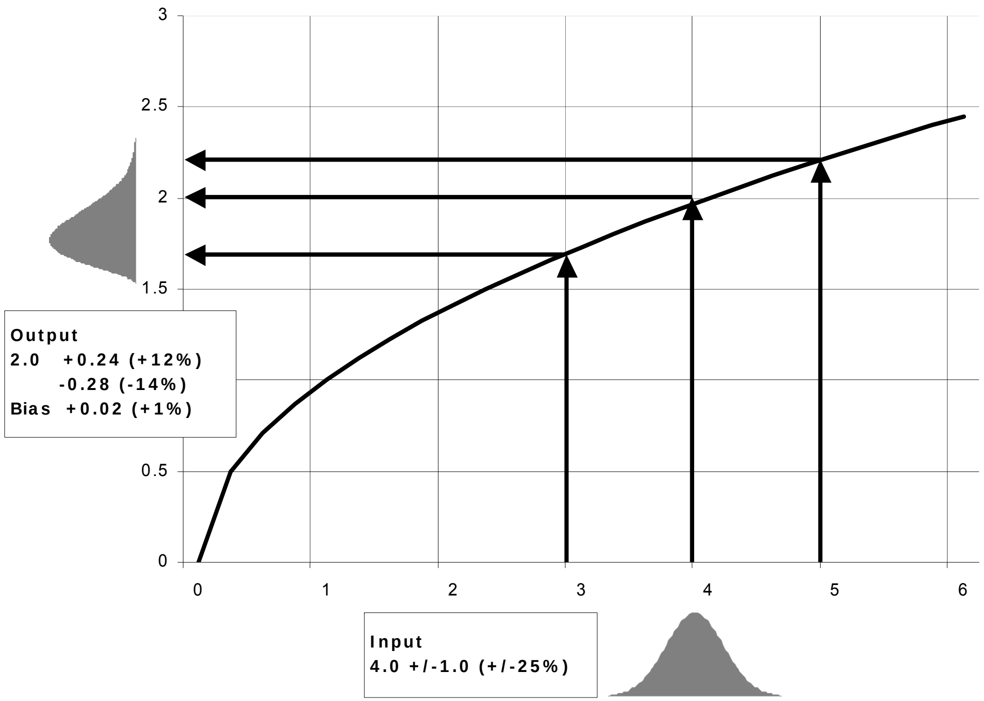
- The overall thermometer measurement uncertainty is due to several sources listed below. Each uncertainty is divided by the coverage factor to find the Standard Uncertainty:
  - $U_{scl}/\sqrt{3}$ scale spacing
  - $U_{plx}/2$ parallax reading error
  - $U_{smk}/2$ scale marking thickness
  - $U_{tol}/2$ manufacturing tolerances
  - $U_{mnc}/2$ meniscus on the top of the mercury column
- Overall measurement uncertainty is found by RSS (Root Sum Square) quadrature combination of uncertainties multiplied by the coverage factor of 2 for a Normal distribution provided the sensitivity for each term is unity:

$$U_t = 2 \times \sqrt{(U_{scl}^2 + U_{plx}^2 + U_{smk}^2 + U_{tol}^2 + U_{mnc}^2)}$$

- RSS is based on the CLT (Central Limit Theorem) whereby combinations of uncertainty distribution will tend toward a Normal distribution illustrated in the following example.

Demo: CLT Combination of Distributions
The demonstration shows that when several distributions are combined the combination is a Normal distribution
This also shows the different distribution and the resultant distribution with standard deviation and the 95% CI

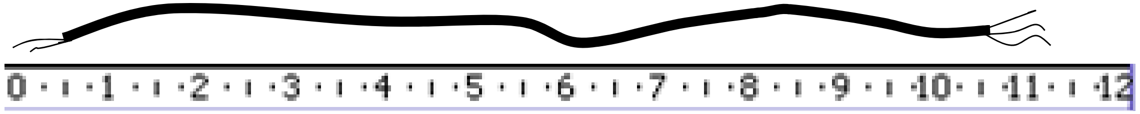
Skewed Distribution



- The flow rate of a Venturi or Orifice meter is proportional to the square root of the differential pressure between the Bore and Throat.
- This non-linear relationship skews the distribution leading to bias in the result which can be corrected by adjusting the mean.

Demo: Square Root Bias; optional

How long is a piece of string (1)



Sources of uncertainty in measuring the length of a piece of string:

String

– Straightness	±0.5"	Normal distribution
– Ends (not frays)	2 x ± 0.1“	Normal distribution
– Elasticity (stretch)	± 0.1"	Normal distribution
– Humidity	± 0.01"	Normal distribution

- Ruler

– Calibration	± 0.01"	Normal distribution
– Resolution (scale)	± 0.25"	Rectangular distribution
– Temperature	± 0.001"	Normal distribution

- Reading

– Parallax error	± 0.125"	Rectangular distribution
– Operator error (not included)		

All uncertainty terms apply to the length of the string with a sensitivity of unity.

How long is a piece of string (2)

$$\sqrt{\left(\frac{0.5}{2}\right)^2 + 2\left(\frac{0.1}{2}\right)^2 + \left(\frac{0.1}{2}\right)^2 + \left(\frac{0.01}{2}\right)^2 + \left(\frac{0.01}{2}\right)^2 + \left(\frac{0.25}{\sqrt{3}}\right)^2 + \left(\frac{0.001}{2}\right)^2 + \left(\frac{0.125}{\sqrt{3}}\right)^2} \cdot 2 = 0.62$$

Quadrature RSS method

- The sources of uncertainty are combined above by the Quadrature method including division by a CF of 2 normal, √3 for rectangular distributions to find the Standard Uncertainty.
- 10.25" ± 0.62" or ± 6.05% with 95% confidence level.
- Dominated by 0.5” string straightness.

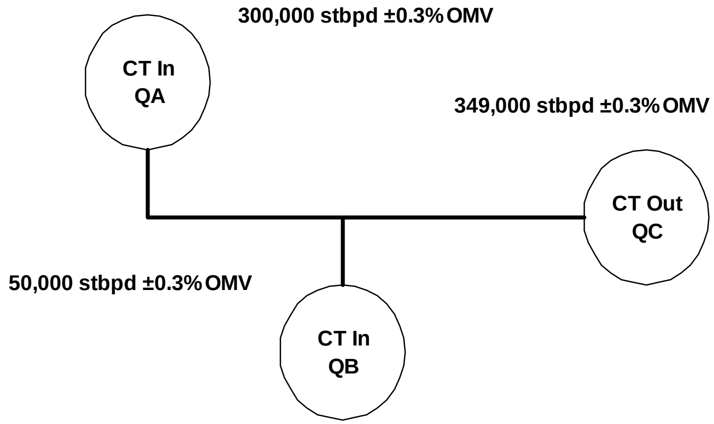
Monte Carlo Simulation method

- Standard Uncertainty is found for each source of uncertainty as RSS and a distribution is generated centered on zero.
- The distributions for each source of uncertainty are added and the Standard Uncertainty found from the standard deviation of the resulting distribution and multiplied by the CF of 2 to given the overall uncertainty with CI of 95%.

Both approaches are demonstrated in the following example.

Demo: How long is a piece of string by Quadrature and MCS.
The demonstration shows that RSS and MCS give the same result for this simple example.

Pipeline Allocation Uncertainty (1)



- A pipeline has two entrants with a single sales discharge all with Custody Transfer metering conforming to OIML R-117 Class 0.3A ±0.3% OMV (Of Measured Value).
- The pipeline imbalance between entrants and sales is -0.3%OMV, which is just on the measurement uncertainty limit.
- Sales are allocated in proportion to each entrants production.
- The allocation procedure will impact each entrants uncertainty exposure differently due to the different rates of production.

Pipeline Allocation Uncertainty (2)

$$y = f(X_1, X_2, ...X_N) \quad (1)$$

$$U = \frac{\sqrt{(\Theta_1 \cdot U_1 \cdot X_1)^2 + (\Theta_2 \cdot U_2 \cdot X_2)^2 + ... + (\Theta_N \cdot U_N \cdot X_N)^2}}{y} \quad (2)$$

$$U = \frac{\sqrt{\begin{array}{l}[y - f[(X_1 - U_1 \cdot X_1), X_2, ...X_N]]^2 ... \\ +[y - f[X_1, (X_2 - U_2 \cdot X_2), ...X_N]]^2 ... \\ + ... \\ +[y - f[X_1, X_2, ...(X_N - U_N \cdot X_N)]]^2\end{array}}}{y} \quad (3)$$

Quadrature Uncertainty Analysis

- For a functional relationship (1)
- The relative uncertainty U_i is multiplied by the measured value to find the absolute uncertainty u_i.
- This is multiplied by the sensitivity Θ_i found from the partial derivatives for each term in the function.
- The overall uncertainty is then found from the square root of the sum of the squares (2).
- The sensitivity can also be found from the function by perturbation of each term by the uncertainty and finding the square root sum of squares (3).

Pipeline Allocation Uncertainty (3)

Pipeline Measurement and Uncertainty

$Q_A := 300000$ $\quad UQ_A := 0.3\%$ $\quad$ Entrant A

$Q_B := 50000$ $\quad UQ_B := 0.3\%$ $\quad$ Entrant B

$Q_C := 349000$ $\quad UQ_C := 0.3\%$ $\quad$ Discharge C

Quadrature uncertainty combination with sensitivity by partial derivitive.

Pipeline entrant A allocation

$$AQ_A := \frac{Q_A \cdot Q_C}{Q_A + Q_B} \qquad AQ_A = 299142.86$$

Entrant A allocation sensitivity terms found by partial differentiation

$$\Theta AQ_{AQA} := \frac{Q_B \cdot Q_C}{(Q_A + Q_B)^2} \qquad \Theta AQ_{AQA} = 0.14 \qquad \frac{\partial}{\partial Q_A}\frac{Q_A \cdot Q_C}{Q_A + Q_B} = 0.14$$

$$\Theta AQ_{AQB} := \frac{-Q_A \cdot Q_C}{(Q_A + Q_B)^2} \qquad \Theta AQ_{AQB} = -0.85 \qquad \frac{\partial}{\partial Q_B}\frac{Q_A \cdot Q_C}{Q_A + Q_B} = -0.85$$

$$\Theta AQ_{AQC} := \frac{Q_A}{Q_A + Q_B} \qquad \Theta AQ_{AQC} = 0.86 \qquad \frac{\partial}{\partial Q_C}\frac{Q_A \cdot Q_C}{Q_A + Q_B} = 0.86$$

Entrant A allocation uncertainty by Quadrature with partial derivitive senstivity terms

$$UAQ_A := \frac{\sqrt{(UQ_A \cdot Q_A \cdot \Theta AQ_{AQA})^2 + (UQ_B \cdot Q_B \cdot \Theta AQ_{AQB})^2 + (UQ_C \cdot Q_C \cdot \Theta AQ_{AQC})^2}}{AQ_A}$$

$UAQ_A = 0.31 \cdot \%$

Pipeline entrant B allocation

$$AQ_B := \frac{Q_B \cdot Q_C}{Q_A + Q_B} \qquad AQ_B = 49857.14$$

Entrant A allocation sensitivity terms found by partial differentiation

$$\Theta AQ_{BQA} := \frac{-Q_B \cdot Q_C}{(Q_A + Q_B)^2} \qquad \Theta AQ_{BQA} = -0.14 \qquad \frac{\partial}{\partial Q_A}\frac{Q_B \cdot Q_C}{Q_A + Q_B} = -0.14$$

$$\Theta AQ_{BQB} := \frac{Q_A \cdot Q_C}{(Q_A + Q_B)^2} \qquad \Theta AQ_{BQB} = 0.85 \qquad \frac{\partial}{\partial Q_B}\frac{Q_B \cdot Q_C}{Q_A + Q_B} = 0.85$$

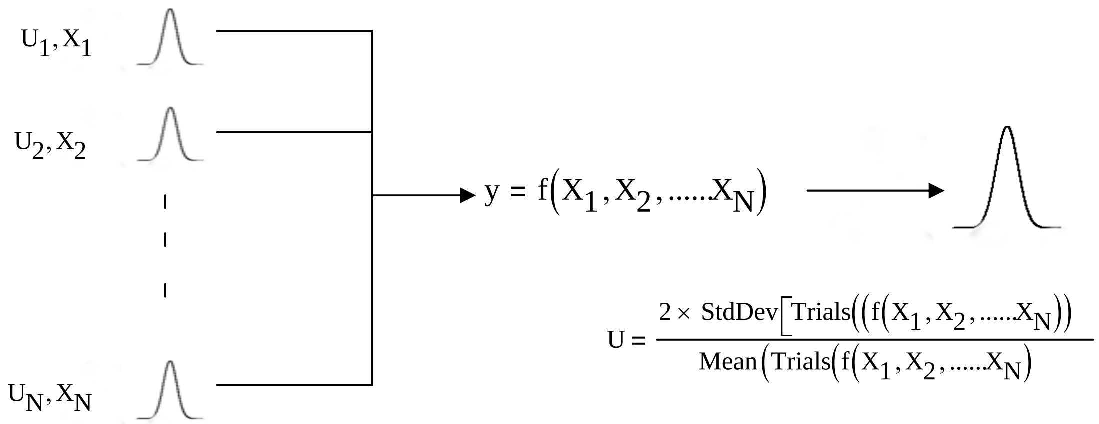
$$\Theta AQ_{BQC} := \frac{Q_B}{Q_A + Q_B} \qquad \Theta AQ_{BQC} = 0.14 \qquad \frac{\partial}{\partial Q_C}\frac{Q_B \cdot Q_C}{Q_A + Q_B} = 0.14$$

Entrant B allocation uncertainty by Quadrature with partial derivitive senstivity terms

$$UAQ_B := \frac{\sqrt{(UQ_A \cdot Q_A \cdot \Theta AQ_{BQA})^2 + (UQ_B \cdot Q_B \cdot \Theta AQ_{BQB})^2 + (UQ_C \cdot Q_C \cdot \Theta AQ_{BQC})^2}}{AQ_B}$$

$UAQ_B = 0.47 \cdot \%$

Pipeline Allocation Uncertainty (4)



$$y = f(X_1, X_2, \ldots\ldots X_N)$$

$$U = \frac{2 \times \mathrm{StdDev}\left[\mathrm{Trials}\left(\left(f(X_1, X_2, \ldots\ldots X_N\right)\right)\right.}{\mathrm{Mean}\left(\mathrm{Trials}\left(f(X_1, X_2, \ldots\ldots X_N\right)\right)}$$

Monte Carlo Simulation Uncertainty

- Distributions with a Standard Uncertainty, found from the measurement uncertainty divided by the CF, are generated and applied to the function.
- This is repeated several thousand times.
- The Standard Uncertainty is found from the standard deviation of the resultant distribution and multiplied by the CF of 2 to find the 95% CI.

RSS with partial derivative and perturbation sensitivity and MCS combined uncertainty

Pipeline Allocation Uncertainty (5)

Measurement Quantity and Uncertainty		
Stream	Flow rate	Uncertainty
Q_A	300,000	0.30%
Q_B	50,000	0.30%
Q_C	349,000	0.30%

Monte Carlo Simulation Uncertainty				
Allocation	Input Trial Qtmcs	Allocation Trials Aqtmcs	Allocation Mean AQmcs	Uncertainty UAQmcs
AQ_{Amcs}	-	#DIV/0!	299,143	0.31%
AQ_{Bmcs}	-	#DIV/0!	49,857	0.47%
Q_C	-			

Quadarature Partial Derivitive Uncertainty				
Allocation	Allocation AQpd	Sensitivity ΘAQ_{Apd}	Sensitivity ΘAQ_{Bpd}	Uncertainty UAQpd
AQ_{Apd}	299,143	0.14	- 0.14	0.31%
AQ_{Bpd}	49,857	- 0.85	0.85	0.47%
Q_C		0.86	0.14	

Quadrature Perturbation Uncertainty				
Allocation	Allocation AQpt	Deviation ΔAQ_{Apt}	Deviation ΔAQ_{Bpt}	Uncertainty UAQpt
AQ_{Apt}	299,143	128.53	- 128.53	0.31%
AQ_{Bpt}	49,857	- 128.26	128.26	0.47%
Q_C		897.43	149.57	

Measurement Quantity and Uncertainty		
Stream	Flow rate	Uncertainty
Q_A	300,000	1.00%
Q_B	50,000	1.00%
Q_C	349,000	0.30%

Monte Carlo Simulation Uncertainty				
Allocation	Input Trial Qtmcs	Allocation Trials Aqtmcs	Allocation Mean AQmcs	Uncertainty UAQmcs
AQ_{Amcs}	-	#DIV/0!	299,138	0.36%
AQ_{Bmcs}	-	#DIV/0!	49,859	1.25%

Uncertainty Method Comparison

Demo: MCS RSS PD & Perturbation.

The demonstration shows that for this simple allocation procedure the results are the same for all three methods.

The uncertainty analysis shows how the smaller entrants uncertainty is disproportionately larger than the larger entrant.

Applications: Oil Standard Volume Uncertainty

Oil Standard Volume Uncertainty						
Fluid	**Quantity**	**Name**	**Unit**	**Value**	**Uncertainty**	
Conditions	Temperature	Tmix	°F	78.00	1.00	0.00
	Pressure	Pmix	psig	250.00	10.00	0.00
Oil	Gravity	APIoil	ºAPI	21.00	0.40	0.00
	Vapour Pressure	Pvap	psig	10.00	2.00	0.0000
Results	Thermal Correction API 11.1	Ctloil	factor	0.992839	0.05%	1.017613
	Pressure correction API 11.2.1	Cploil	factor	1.001028	0.05%	1.000000
	Volume Correction Factor	VCFoil	factor	0.993860		1.017613
	Volume	Qvline	bpd	50,000	0.20%	-
	Standard Volume	Qvstd	stbpd	49,693	0.22%	-

Demo: Oil Standard Volume Uncertainty

API Chpt 11.1 oil thermal correction and API Chpt 11.2.1 oil compressibility correction.

Uses MCS to calculate complex correction factors including the method uncertainty.

Applications: Gas Density Uncertainty

AGA8 Gas Density						
Line Conditions	**Measurement**	**Uncertainty**				**Trial Values**
Temperature deg C	25.00	0.450				0.00
Pressure bara	18.00	0.755				0.00
Gas Composition	**Compostion mol%**	**Normalised mol%**	**Component Uncertainty %**	**Uncertainty mol%**	**Trials**	**Normalised Trials**
Nitrogen mol%	0.720	0.720	1.00%	0.0072	0.0000	#DIV/0!
Carbon Dioxide mol%	1.360	1.360	1.00%	0.0136	0.0000	#DIV/0!
Methane mol%	85.330	85.330	2.00%	1.7066	0.0000	#DIV/0!
Ethane mol%	6.150	6.150	1.00%	0.0615	0.0000	#DIV/0!
Propane mol%	3.810	3.810	1.00%	0.0381	0.0000	#DIV/0!
n-Butane mol%	2.020	2.020	1.00%	0.0202	0.0000	#DIV/0!
i-Butane mol%	0.000	0.000	1.00%	0.0000	0.0000	#DIV/0!
n-Pentane mol%	0.580	0.580	1.00%	0.0058	0.0000	#DIV/0!
i-Pentane mol%	0.000	0.000	1.00%	0.0000	0.0000	#DIV/0!
n-Hexane mol%	0.030	0.030	1.00%	0.0003	0.0000	#DIV/0!
n-Heptane mol%	0.000	0.000	1.00%	0.0000	0.0000	#DIV/0!
n-Octane mol%	0.000	0.000	0.00%	0.0000	0.0000	#DIV/0!
n-Nonane mol%	0.000	0.000	0.00%	0.0000	0.0000	#DIV/0!
n-Decane mol%	0.000	0.000	0.00%	0.0000	0.0000	#DIV/0!
Total mol%	100.000	100.00			0.00	#DIV/0!
Normalised	**True Result**	**Method Uncertainty**	**MCS Mean**	**MCS Uncertainty**	**Trials with Method**	**Trials**
Line Density Kg/m3 (AGA8)	14.97	0.10%	14.97	4.44%	#VALUE!	#VALUE!
Standard Density Kg/m3 (AGA8)	0.8311	0.10%	0.8311	0.34%	#VALUE!	#VALUE!
Line/Standard	18.01		18.01	4.43%	#VALUE!	#VALUE!

Demo: AGA8 Density Uncertainty

Uses MCS to with the AGA8 Equation of State method.

Automatically takes account of dependency between inputs due to gas composition normalisation and within AGA8.

Generic Uncertainty Model Simulation (GUMS)

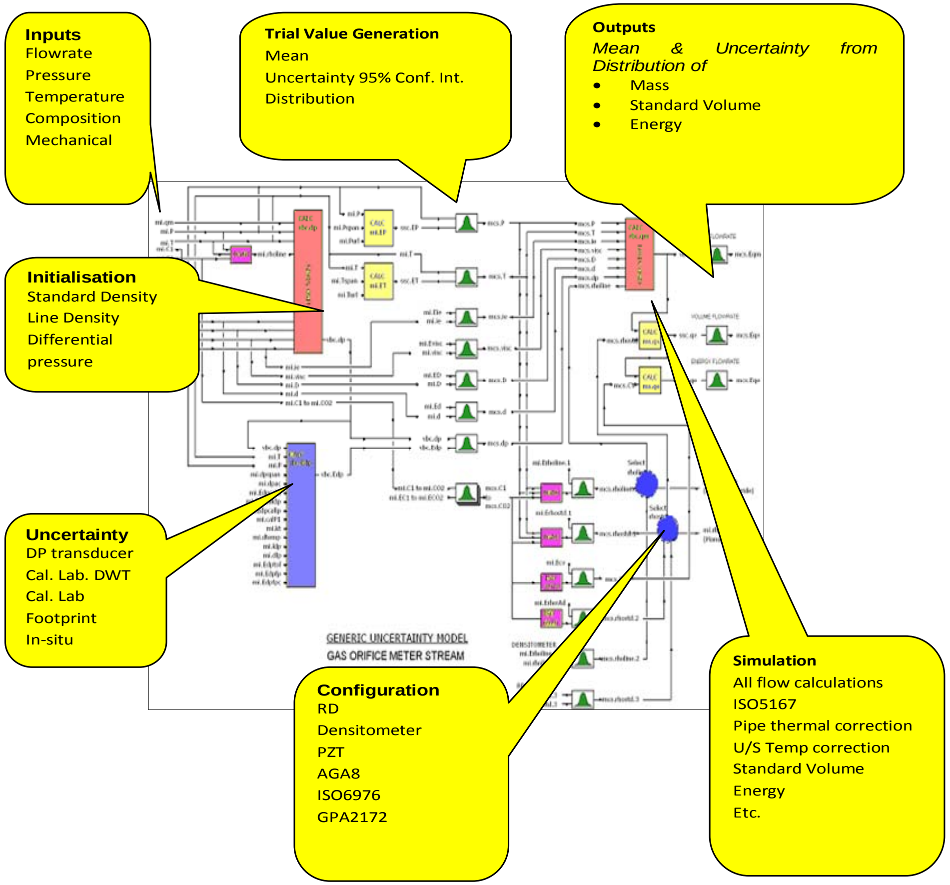
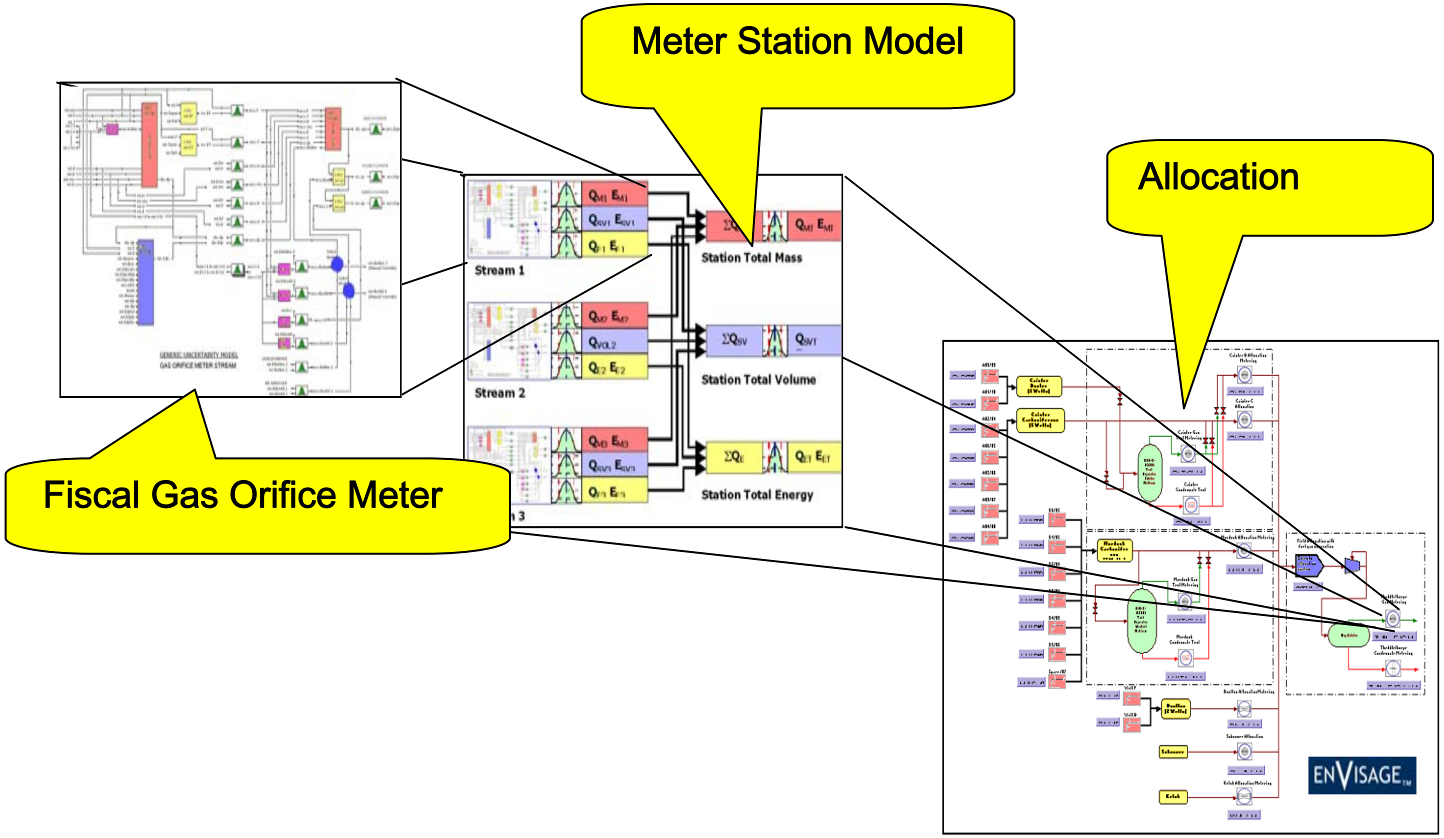


Figure Gas Custody Transfer Meter

Propagation of Uncertainty with MCS



Conclusions

- Uncertainty
  - Validated the Central Limit Theorem with MCS.
  - Showed that non-linearity of a function can lead to bias.
- Combination of Uncertainties
  - Compared RSS and MCS in “How long is apiece of string” and got the same results which also confirmed the validity of the $\sqrt{3}$ CF for uniform distributions.
  - Demonstrated with the “Pipeline Allocation Uncertainty” that both RSS and the MCS methods all give the same result.
  - Showed how the Custody Transfer meter uncertainty is not a good indication of the final allocation uncertainty such that every case must be looked at in case there is excessive uncertainty exposure due to allocation.
- Applications
  - Demonstrated how complex methods can be correctly dealt with by MCS and that dependency between inputs within the method is correctly handled.
  - Showed how uncertainty distributions can be propagated indefinitely through all stages of allocation and data processing.

Thank you for listening

Questions?